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A
BRIEF COURSE
IN
ARITHMETIC,
ORAL AND WRITTEN.

ON THE BASIS OF WORKS

BY BENJAMIN GREENLEAF, A.M.

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PREFACE.

THIS inductive book has been made to meet demands which have grown with the advance of educational ideas.

Young pupils, who are expected to finish a course of grammar-school study and who are to be trained in the lower grades to facility and accuracy in the fundamental use of numbers, require training in both oral and written work. While they are too immature to understand the *theory* and *science* of numbers, they may be especially benefited by much simple *practice.* For them the book furnishes, what is desirable, much practical work, and little theory.

There are many learners, whose circumstances compel them to leave school at an early age. They have little time to spend on definitions and theory, but want practice. This work will help such to acquire the ability to use numbers and apply them to the ordinary transactions of life.

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A BRIEF COURSE

IN

ARITHMETIC.

NOTATION AND NUMERATION.

1. A **Unit** is a single thing, or one; as one book, one slate.

2. A **Number** is a unit, or a collection of units; as one book, five slates.

3. **Arithmetic** treats of numbers and their use.

4. **Figures** are characters used to express numbers.

5. Ten different figures are used in writing numbers:

Name.	**Zero,**	**One,**	**Two,**	**Three,**	**Four,**	**Five,**	**Six,**	**Seven,**	**Eight,**	**Nine.**
Figure.	**0,**	**1,**	**2,**	**3,**	**4,**	**5,**	**6,**	**7,**	**8,**	**9.**

These figures used alone express the number of units shown by their names.

The zero, or cipher, used alone expresses *no* units.

6. To express numbers larger than nine requires the use of two or more figures.

A figure used alone has only a *simple name* and value, but a figure used with other figures has also a *place name* and value.

One Ten.

7. In naming numbers nine ones and one are called a **Ten.**

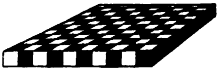
In expressing tens, or tens and ones, two figures are written side by side, the figure on the right denoting ones, and that on the left denoting tens. Thus,

1 ten, or 1 ten 0 ones, in figures is 10
2 tens and 3 ones, or twenty-three, in figures is . . 23
4 tens and 5 ones, or forty-five, in figures is 45

and so on; in these numbers the figures 1, 2, and 4 have, respectively, the place names and value of 1 ten, 2 tens, and 4 tens.

8. In reading a number composed of tens and ones, the tens and ones are read together as so many ones. Thus,

83 is read eighty-three ones, or briefly eighty-three.



One Hundred.

9. In naming numbers 10 tens are called a **Hundred.**

In expressing hundreds, or hundreds and tens, or hundreds, tens, and ones, three figures are written side by side, the first figure on the right expressing ones; the second, tens; and the third, hundreds. Thus,

1 hundred, or 10 tens, in figures is 100
2 hundred, or 20 tens, in figures is 200
3 hundred twenty-one, in figures is 321

10. In reading a number composed of hundreds, tens, and ones, the hundreds, tens, and ones are read together as so many ones. Thus,

472 is read four hundred seventy-two ones, or briefly four hundred seventy-two.

EXERCISES.

Read the following numbers:

1.	**2.**	**3.**	**4.**	**5.**
12	*812*	*403*	*101*	*499*
44	*41*	*312*	*397*	*786*
6	*660*	*707*	*635*	*652*
80	*818*	*900*	*63*	*496*
20	*292*	*501*	*919*	*999*

Write in figures arranged in columns:

6. The numbers between sixty-four and seventy-four.

7. Twenty-seven; ninety; nineteen; fifty-six; forty-one; eight.

8. The numbers between one hundred and one hundred twelve.

9. The numbers between one hundred nine and one hundred twenty.

10. The numbers between one hundred eighty and two hundred.

11. Three hundred; three hundred three; three hundred thirty; three hundred thirty-three.

12. Four hundred nine; four hundred five; four hundred ninety; nine hundred fifty-nine.

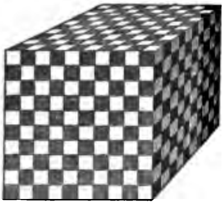
13. Seven hundred seventy-seven; two hundred twenty-two; eight hundred eighty-eight; six hundred seven.

14. Five hundred two; five hundred ninety-nine; four hundred seventy-one; nine hundred ninety-eight.

15. Six hundred two; eight hundred thirty-three; four hundred forty-four.

THOUSANDS.

11. In naming numbers 10 hundreds are called a **Thousand.**



One Thousand.

16. In the picture, how many small blocks in one layer of the large block?

17. How many layers of one hundred small blocks are there in the large block?

18. How many small blocks are there in two layers of the large block? In three layers? In ten layers?

19. Count by hundreds from one hundred to one thousand.

12. In writing numbers the figure at the left of that denoting hundreds expresses thousands. Thus,

1 thousand in figures is 1000
2 thousand in figures is 2000
5 thousand 6 hundred in figures is..... 5600

EXERCISES.

Read the following numbers:

20.	**21.**	**22.**	**23.**
7000	*9300*	*1301*	*8243*
8000	*4050*	*3708*	*5005*
2600	*6224*	*5060*	*6024*
8108	*4579*	*7802*	*9999*

Write in figures arranged in columns:

24. Six thousand one hundred twenty.
25. Eight thousand four hundred one.
26. One thousand one hundred sixty-eight.
27. Three thousand three hundred three.
28. Seven thousand seventy-nine.
29. Five thousand fifty-six.
30. Nine thousand eight hundred sixty-four.

ORDERS OF UNITS.

13. Orders of Units are the successive units formed by taking together ten ones, ten hundreds, ten thousands, and so on. Thus,

Ones are units of the *first order ;*
Tens are units of the *second order ;*
Hundreds are units of the *third order ;*
Thousands are units of the *fourth order ;*

and so on.

14. In numbers expressed in figures, the successive orders of units are numbered, beginning with the ones' figure. Thus,

A figure in ones' place denotes units of the first order,
A figure in tens' place denotes units of the second order,
A figure in hundreds' place denotes units of the third order,
A figure in thousands' place denotes units of the fourth order, and so on.

31. In 365, which figure expresses ones? which tens?

32. In 7436, which figure expresses hundreds? which thousands?

33. In 9278, which figure expresses units of the second order? which units of the fourth order?

15. 10 thousands form a unit of the fifth order, called a **Ten-thousand.**

16. In a number the figure at the left of thousands, or in the fifth place, denotes ten-thousands. Thus,

1 ten-thousand, or 10 thousands, in figures is . . . 10000
2 ten-thousands, or 20 thousands, in figures is . . 20000
3 ten-thousands, or 30 thousands, in figures is . . 30000
and so on.

34. In 32465, which figure expresses thousands? which ten-thousands?

35. In 47809, which figure expresses units of the fourth order? which units of the fifth order?

17. 10 ten-thousands form a unit of the sixth order, called a **Hundred-thousand.**

18. In a number the figure at the left of ten-thousands, or in the sixth place, denotes hundred-thousands. Thus,

1 hundred-thousand in figures is 100000
2 hundred-thousands in figures is.... 200000
3 hundred-thousands in figures is.... 300000

and so on.

36. In 368000, which figure expresses ten-thousands? which hundred-thousands?

37. In 537410, which figure expresses units of the fifth order? which units of the sixth order?

19. In reading a number composed of five orders of units, the fourth and fifth are read together as thousands.

Thus, 63407 is named and read sixty-three thousand four hundred seven.

20. In naming and reading a number composed of six orders of units, the fourth, fifth, and sixth are read together as thousands.

Thus, 791623 is named and read seven hundred ninety-one thousands six hundred twenty-three.

21. A **Group,** or **Period,** is each successive three orders of units, beginning with ones.

The *Group of Ones* is composed of ones, tens, and hundreds.

The *Group of Thousands* is composed of ones, tens, and hundreds of thousands.

The comma (,) may be used to mark off the groups in a number expressed in figures.

Thus, 791623 may be written 791,623.

38. In 503,421 which figures make the ones' group?

39. In 718,300 which figures make the thousands' group?

EXERCISES.

Write and read:

40.	**41.**	**42.**	**43.**
51,000	*20,600*	*301,000*	*702,606*
63,440	*72,045*	*500,000*	*834,621*
79,367	*99,999*	*115,053*	*999,099*

Write in figures:

44. Ten thousand three hundred twenty-nine.

45. Twenty-two thousand four hundred three.

46. Sixty thousand two hundred forty-seven.

47. Forty-seven thousand.

48. Fifty-eight thousand seven hundred one.

49. Ninety-five thousand eleven.

50. One hundred thirty-seven thousand.

51. Four hundred fifty-one thousand six hundred sixty-six.

52. Eight hundred thousand.

53. Seven hundred fifty thousand two hundred ninety-nine.

DECIMALS.

22. Decimals are numbers written at the right of the ones' place.

23. Decimals are distinguished from the ones' figure, by being at the right of a point (.), called the *decimal point.*

The first order at the right of the point is *tenths,*

The second order at the right of the point is *hundredths,*

The third order at the right of the point is *thousandths,*

and so on. Thus,

0.5	is read	5 tenths,
0.17	"	17 hundredths,
0.075	"	75 thousandths,
2.05	"	2, and 5 hundredths,
15.605	"	15, and 605 thousandths.

NOTE. In reading a number, use *and* only between the ones and the decimal to indicate the decimal point.

EXERCISES.

Read,

54. *0.125*
55. *6.05*
56. *1.103*
57. *3.008*
58. *11.07*
59. *1.107*
60. *0.06*
61. *110.7*
62. *67.58*
63. *657.5*
64. *119.15*
65. *1.003*

Write in figures:

66. One hundred, and one tenth.
67. Sixteen, and eighty-seven hundredths.
68. Three, and sixteen thousandths.
69. One hundred ninety-seven thousandths.
70. Seven, and three thousandths.
71. Nine hundred, and nine tenths.
72. Thirty-two, and thirty-two hundredths.
73. Eight hundred thirteen thousandths.
74. Eleven, and one hundred eleven thousandths.

Writing and Reading Numbers.

24. **Notation** is writing numbers in figures.

25. **Numeration** is reading numbers written in figures.

26. The method of writing numbers, the successive orders of units, and the names of groups are shown in the following

NUMERATION TABLE.

NAMES OF ORDERS OF UNITS.	Hundred-millions,	Ten-millions,	*Millions,*	Hundred-thousands,	Ten-thousands,	*Thousands,*	Hundreds,	Tens,	*Ones,*	Tenths,	Hundredths,	*Thousandths.*
NUMBER.	1	5	4 ,	2	0	5 ;	6	3	8 .	2	5	7
NAMES OF GROUPS.	Millions.			Thousands.			Ones.			Thousandths.		

The figures in the table express one hundred fifty-four million two hundred five thousand six hundred thirty-eight, and two hundred fifty-seven thousandths.

27. The last place-name in each group is the group-name. The groups above millions are, billions, trillions, etc.; below thousandths, millionths, billionths, etc.

28. A Principle of Notation.

Ten units of any order equal a unit of the order next larger.

EXERCISES.

75. Write and read 25362795.

Solution.—25362795 marked off into groups becomes 25,362,795; the group at the right expresses 795 ones, or 795; the next group expresses 362 thousands; and the third expresses 25 millions.

The whole is read: twenty-five million three hundred sixty-two thousand seven hundred ninety-five.

76. Write and read 87409.

77. Write and read 106789.

78. Write and read 2125.79.

Solution.—2125.79 marked off into groups becomes 2,125.79, which is read two thousand one hundred twenty-five, and seventy-nine hundredths.

79. Write and read 604.038.

80. Write and read 67445.8.

81. Write in figures, seven million six hundred ninety-three thousand two hundred four, and two hundred three thousandths.

Solution.—Writing 7 for the millions, 693 for the thousands, 204 for the ones, and 203 for the thousandths, we have 7,693,204.203, as the number required.

82. Write in figures eighty-seven thousand four hundred nine.

29. Rule for Reading Numbers.

Beginning at the ones' place separate the number, by commas, into as many groups as possible of three figures each.

Begin at the left and read each group as if it stood alone, giving the group-name except to the ones' group.

If there is a decimal, read it as if it were at the left of the point, adding the place-name of the last figure.

Write and read:

83. *456*
84. *1385*
85. *7075*
86. *12001*
87. *199*
88. *78382*
89. *315620*
90. *445579*
91. *6004*
92. *73241*
93. *132475*
94. *300002*
95. *21005*
96. *73121*
97. *3880*
98. *6321456*
99. *0.31*
100. *17.03*
101. *0.016*
102. *1.039*
103. *14666.2*
104. *1965.8*
105. *35.06*
106. *1860.007*

30. Rule for Writing Numbers in Figures.

Beginning at the left write the figures of each group in their order, filling vacant places and groups with ciphers.

Write in figures:

107. Five hundred fifty.

108. One thousand six hundred thirty-five.

109. Eleven thousand one.

110. Twelve hundred twelve.

111. Sixty-three thousand five hundred ninety-two.

112. Seventy-three thousand nine hundred nine.

113. One hundred forty thousand five hundred sixteen.

114. Fifty-one thousand nineteen.

115. Three hundred thousand four.

116. Nine thousand seven hundred nine.

117. Seventeen thousand two hundred eleven.

118. Eighty-eight thousand eighty-three.

119. Sixty-nine thousand seven hundred five.

120. Twenty-two thousand one hundred forty-one.

121. One hundred thousand sixty-seven.

122. One million one thousand one.

123. Seven hundred thirty thousand four hundred twenty-eight.

124. One, and thirteen hundredths.

125. Fifteen, and eight thousandths.

126. One hundred fifty-two thousandths.

127. Six hundred ninety-two thousandths.

128. Five hundred fifty-one, and eight tenths.

129. Four thousand five hundred nine, and eighteen hundredths.

130. Nine hundred seventy-five thousandths.

131. Five hundred sixty-six, and forty-one hundredths.

132. Thirty-seven thousand six hundred, and three thousandths.

133. Fifty thousand five, and five tenths.

134. One hundred one thousand, and sixteen hundredths.

135. Three hundred twenty-four thousand, and twenty-five thousandths.

136. One million sixteen thousand four hundred five, and seventy-five hundredths.

137. Five hundred nineteen thousand five hundred twenty-nine, and five tenths.

138. Three million four hundred eleven thousand nine, and one hundred sixty-seven thousandths.

QUESTIONS.

1. What is a unit? 2. What is a number? 3. Of what does Arithmetic treat?

4. What are figures? 5. How many figures are used in writing numbers?

7. In naming numbers what are nine and one called? 9. What are ten tens called? 11. What are ten hundreds called?

21. What is a group? Of what is the group of ones composed? The group of thousands?

22. What are decimals? 23. When written in figures how are they distinguished from ones? What is the first order at the right of the point called? The second order? The third?

24. What is notation? 25. What is numeration? 26. Give the groups of the numeration table. The orders of units of each group?

28. What is a principle of notation?

29. How do you separate a number in figures for reading? How do you read each group?

30. How do you write numbers in figures?

ADDITION.

31. 1. The side of a house has 5 windows and the end 7. How many windows are 5 windows and 7 windows?

2. If James has 4 books and his brother 3, how many have both? How many are 4 and 3?

3. How many cents are 6 cents and 2 cents? How many are 6 and 2?

4. How many trees are 5 trees and 3 trees? How many are 5 and 3?

5. How many marbles are 4 marbles and 4 marbles? How many are 4 and 4?

6. How many boys are 5 boys and 4 boys? How many are 5 and 4?

7. If you had 5 apples and John should give you 3, how many would you have? How many are 5 and 3?

8. Five peaches and 3 more are how many peaches? How many are 5 and 3?

9. Mary has 5 books and Lucy 5. How many books have both?

10. In one carriage are 5 boys and in another 4. How many boys in both? How many are 5 and 4?

11. In one pasture there are 6 sheep and 2 in another. How many sheep in both?

12. Henry wrote one day 6 lines and 4 the next day. How many lines did he write both days? How many are 6 and 4?

13. How many girls are 6 girls and 5 girls? How many are 6 and 5?

14. How many dollars are 6 dollars and 3 dollars? What is one of 6 dollars? One of 3 dollars? What then is a unit of each?

32. **Like Numbers** are numbers with the same kind of unit. Thus,

7, 5, 3; or 6 cents, 2 cents, 9 cents are like numbers.

33. **Addition** is finding a number equal to two or more given numbers.

Only like numbers can be added.

34. The **Sum** or **Amount** is the result of an addition.

35. The **Sign of Addition** is +. It is named *plus* and means *more.* It is generally read *and.*

Thus, $4 + 5$ is read 4 and 5.

36. The **Sign of Equality** is =. It means *equal* or *equal to.*

Thus, $5 + 6 = 11$ is read 5 and 6 are 11.

37. The **Dollar Sign, $,** means *dollars.*

Thus, $8 may be read eight dollars.

Copy and complete the following

ADDITION TABLES.

15.

1 + 1 =
3 + 1 =
2 + 1 =
4 + 1 =
7 + 1 =
6 + 1 =
5 + 1 =
9 + 1 =
8 + 1 =

16.

1 + 2 =
4 + 2 =
2 + 2 =
3 + 2 =
6 + 2 =
7 + 2 =
8 + 2 =
5 + 2 =
9 + 2 =

17.

1 + 3 =
3 + 3 =
2 + 3 =
5 + 3 =
4 + 3 =
7 + 3 =
6 + 3 =
9 + 3 =
8 + 3 =

18.

1 + 4 =
4 + 4 =
3 + 4 =
2 + 4 =
5 + 4 =
7 + 4 =
6 + 4 =
8 + 4 =
9 + 4 =

19.

1 + 5 =
3 + 5 =
2 + 5 =
4 + 5 =
6 + 5 =
5 + 5 =
7 + 5 =
9 + 5 =
8 + 5 =

20.

1 + 6 =
5 + 6 =
3 + 6 =
2 + 6 =
4 + 6 =
6 + 6 =
8 + 6 =
7 + 6 =
9 + 6 =

21.

2 + 7 =
1 + 7 =
3 + 7 =
4 + 7 =
6 + 7 =
5 + 7 =
8 + 7 =
7 + 7 =
9 + 7 =

22.

3 + 8 =
2 + 8 =
1 + 8 =
5 + 8 =
4 + 8 =
6 + 8 =
9 + 8 =
7 + 8 =
8 + 8 =

23.

1 + 9 =
3 + 9 =
4 + 9 =
2 + 9 =
6 + 9 =
7 + 9 =
5 + 9 =
8 + 9 =
9 + 9 =

24. Add 2 and 4. 5 and 1. 4 and 3. 3 and 7.
25. Add 8 and 2. 6 and 5. 9 and 6. 3 and 8.
26. Add 3 and 5. 7 and 7. 8 and 5. 4 and 9.
27. Add 7 and 7. 3 and 3. 5 and 9. 6 and 7.
28. Add 9 and 2. 5 and 7. 8 and 8. 2 and 6.
29. Add 8 and 8. 4 and 4. 3 and 9. 4 and 5.
30. Add by 2's from 1 to 27.

Solution. — 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27.

Add

31. By 2's from 2 to 30.
32. By 3's from 0 to 33.
33. By 4's from 5 to 29.
34. By 5's from 6 to 38.
35. By 6's from 1 to 37.
36. By 6's from 3 to 39.
37. What is the sum of 2 + 4 + 3? Of 4 + 3 + 2?
38. What is the sum of 5 + 2 + 6? Of 6 + 5 + 2?

38. A Principle of Addition.

The sum of numbers is the same in whatever order they may be added.

WRITTEN EXERCISES.

Copy and add

39.

$$\begin{array}{r} 5 \\ 4 \\ 3 \\ 2 \\ \hline 14 \end{array}$$

Solution. — Begin at the bottom, and add upwards, naming results, thus: 2, 5, 9, 14; sum 14, which we write under the numbers added.

To test the work, begin at the top, and add downwards, thus: 5, 9, 12, 14; sum as before, 14.

40.	**41.**	**42.**	**43.**	**44.**	**45.**	**46.**
3	2	6	3	6	4	6
3	4	2	2	5	4	4
4	3	3	1	4	3	5
5	5	2	5	3	5	1
1	4	5	6	3	5	6

47.	**48.**	**49.**	**50.**	**51.**	**52.**
30	20	60	300	400	300
30	40	20	200	500	200
40	50	30	100	300	500
50	50	40	500	600	400
10	40	10	600	200	100
160			1700		

In Exercise 47 there being no ones in the column of ones, we write 0 in the ones' place in the sum; and in Exercise 50, there being no ones in the column of ones and no tens in the column of tens, we write 0 in the ones' place, and 0 in the tens' place, in the sum.

53. What is the sum of 236, 541, and 102?

236
541
102
879

Solution. — For convenience, we write the numbers so that units of the same order may be in the same column.

We add, beginning with ones, thus: 2, 3, 9; sum, 9, which we write under the line in ones' place.

Adding the tens, 0, 4, 7; sum, 7 tens, which we write under the line in tens' place.

Adding then the hundreds; 1, 6, 8; sum, 8 hundreds, which we write under the line in hundreds' place.

The sum, then, is 8 hundreds 7 tens 9 ones, or 879.

54. What is the sum of 216, 103, and 520?

55. What is the sum of 452, 123, and 211?

56. What is the sum of 501, 120, and 17?

ORAL EXERCISES.

57. John has 4 apples, Arthur 3, and Henry 2. How many have they in all?

58. Paid for paper 8 cents, and for pencils 6. How much was paid for the whole?

59. James caught 7 fishes, Ernest 5, and Albert 6. How many were caught by them all?

60. Gave $9 for coal, and $11 for wood. How many dollars were given for both?

61. If a writing-book costs 10 cents, a pen 5 cents, and a slate 9 cents, how much do they all cost?

62. A farmer had 27 cows and bought 8 more. How many had he then?

63. Jane has 8 books at school, 7 at home, and has loaned 6. How many has she in all?

64. In one pasture are 10 sheep, in another 7, and in a third 8. How many are there in all?

65. A clothier sold a hat for $5, a vest for $9, and a coat for as much as he received for both the hat and vest. For how much was the coat sold?

66. A boy gave 10 cents for a pencil, 5 cents for a piece of rubber, and 4 cents for a penholder. How much did he pay for all?

67. How many are 17 and 4? 27 and 4? 37 and 4? 67 and 4?

68. Begin with 5 and count by 5's to 65.

69. A man bought a cow for $35, a sheep for $8, and had $6 left. How many had he at first?

70. How many are 11, 7, and 9? 13, 4, and 3? 14, 5, and 8? 31, 6, and 7?

71. Jane had 19 cents and her father gave her 8. How many then had she?

72. In a garden are 6 cherry trees, 8 peach trees, and 12 pear trees. How many trees are there in the garden?

WRITTEN EXERCISES.

73. What is the sum of 595, 361, and 723?

$$\begin{array}{r} 595 \\ 361 \\ 723 \\ \hline 1679 \end{array}$$

Solution. — We write the numbers so that units of the same order may be in the same column.

We add, beginning with ones, thus: 3, 4, 9; sum, 9 ones, which we write under the line in ones' place.

We add the tens, thus: 2, 8, 17; sum, 17 tens, which are 170, or 1 hundred 7 tens. We write under the line the 7 tens in the place of tens, and add the 1 hundred with the hundreds in the next column.

We add the hundreds, thus: 1, 8, 11, 16; sum, 16 hundreds, which are 1600, or 1 thousand 6 hundred. We write under the line the 6 hundreds in hundreds' place and the 1 thousand in thousands' place.

The sum is 1 thousand 6 hundreds 7 tens 9 ones, or 1679.

74. What is the sum of 691, 10, and 9 ?

75. What is the sum of 334, 160, and 123 ?

Write and add —

76.	**77.**	**78.**	**79.**	**80.**
610	717	491	416	929
291	812	870	725	442
208	402	843	240	612

81.	**82.**	**83.**	**84.**	**85.**
821	723	391	131	818
333	651	61	615	98
998	689	781	23	3

86.	**87.**	**88.**	**89.**	**90.**
100	82	402	731	928
99	491	40	617	713
16	149	449	100	56

91.	**92.**	**93.**	**94.**
1781	606	425	2020
121	12	39	451
642	871	620	7770
2401	462	510	1223

ORAL EXERCISES.

95. How many are 13 and 4 ? 23 and 4 ? 63 and 4 ? 93 and 4 ? 15 and 6 ? 35 and 6 ? 6 and 55 ?

96. How many are 11 and 7? 21 and 7? 51 and 7? 71 and 7? 12 and 3? 32 and 3? 83 and 3?

97. How many are 16 and 2? 46 and 2? 46 and 5? 66 and 5? 96 and 5? 17 and 3? 37 and 3?

98. How many are 18 and 7? 28 and 7? 68 and 7? 98 and 7? 14 and 7? 27 and 7?

99. How many are 19 and 8? 29 and 8? 69 and 8? 89 and 9? 10 and 9? 20 and 9? 80 and 9?

100. Count by 6's from 12 to 48. By 7's from 21 to 63.

101. How many are 9, 11, and 8? 14, 6, and 7? 23, 8, and 1? 32, 7, and 9?

102. James solved in the forenoon 13 problems, in the afternoon 8, and in the evening 6. How many did he solve in all?

103. Add by 3's from 1 to 37. By 9's from 9 to 81.

104. A man bought a harness for $31, a bridle for $8, and had $9 left. How many dollars had he at first?

105. How many are 13 + 6 + 4 + 9?

106. Sold a cow for 35 dollars, a sheep for 7, and a calf for 6. How much did they all bring?

107. Mary had 47 apples; Jane gave her 11, and William 9. How many had she then?

108. In a garden are 30 peach-trees, 7 pear-trees, and 9 apple-trees. How many trees are there in the garden?

109. How many are 33 and 29?

110. How many are 23 and 47 ? 31 and 59 ?

111. A farmer has sheep in two folds ; in one are 27 and in the other are 35. How many are there in both folds ?

112. How many are 25 and 35 ? 35 and 25 ?

113. How many are 20 + 23 + 16 ? 16 + 23 + 20 ?

114. A lady paid $55 for a carpet, $20 for a chair, and $15 for a table. How much did she pay for all ?

39. In the **Currency** of the United States, as 100 cents are one dollar, and 10 mills are one cent, cents are hundredths and mills are thousandths of a dollar; so written, cents may occupy two decimal places, tenths and hundredths, and mills one place, thousandths. Thus,

$15.50 is read fifteen dollars fifty cents, and $28.005 is read twenty-eight dollars five mills.

WRITTEN EXERCISES.

115. What is the sum of $12.635, $15.43, and $7.50 ?

$$\begin{array}{r} \$\,12.675 \\ 15.39 \\ 7.50 \\ \hline \$\,35.565 \end{array}$$

Solution. — Writing the numbers so that units of the same order may be in the same column, we begin with thousandths to add, thus : 5 thousandths ; which we write under the line in thousandths' place.

We add the hundredths, thus : 9, 16 hundreths, or 1 tenth 6 hundredths. We write under the line the 6 hundredths in the place of hundredths, and add the 1 tenth with the tenths of the next column.

We add the tenths, thus: 1, 6, 9, 15 tenths, or 1 one and 5 tenths. We write under the line the 5 tenths in the place of tenths, and add the 1 with the ones of the next column.

We add the ones, thus: 1, 8, 13, 15 ones, or 1 ten and 5 ones. We write under the line the 5 ones in the place of ones, and add the 1 ten with the tens of the next column.

We add the tens, thus: 1, 2, 3 tens, which we write under the line in the place of tens.

The sum is $35.565, or 35 dollars 56 cents 5 mills.

Copy and add —

116.	**117.**	**118.**	**119.**
$55.05	$26.	$1614.	$6.125
13.50	37.65	719.	3.81
19.75	42.11	14.	0.625
0.81	0.93	9.	0.005

120.	**121.**	**122.**	**123.**
1234	7261	2168	2341
5610	345	1681	5123
1432	69	2002	4402

40. Rule for Addition.

Write the numbers so that units of the same order may be in the same column.

Add the right-hand column, writing the units of the sum underneath, and adding the tens, if any, to the next column.

So proceed with all the columns, writing the entire sum of the last column.

41. PROOF. — Add the numbers a second time, and in a different order (Art. 38).

124. What is the sum of 9401, 1105, and 47?

125. What is the sum of 424 + 643 + 301 + 46?

126. What is the sum of 6684 + 240 + 7006 + 309 + 1824?

127. 470 + 640 + 126 + 123 + 11 = how many?

128. 136 + 320 + 708 + 17 + 3 = how many?

129. Find the sum of 30, 104, 3047, and 57183.

130. Find the sum of 116.45, 18.94, 27.98, and 3.46.

131. What is the sum of $43.50, $9, $13.50, and $0.85?

132. What is the sum of 150.015, 1.601, 100.11, and 0.717?

133. Paid for a cart $67.15, for having it painted $6.35, and for some repairs upon it $5. For how much must it be sold to gain $2?

134. I have bought six lots of coal, containing 210, 317, 256, 610, 518, and 311 tons, respectively. How many tons are there in all?

135. Bought a barrel of flour for $9.50, sugar for $3, pork for $13.93, corn for $18.50, and a fish for $0.49. What was the cost of the whole?

136. Paid for a farm $11,500, for having it fenced $635.50, and for having a barn built $510.75. What did the whole cost?

137.	**138.**	**139.**	**140.**
31141	4310	6451	148081
6131	709	3723	27305
571	51	4540	11941
450	600	5172	31231
19	173	425	4207

141.	**142.**	**143.**	**144.**	**145.**
$63.21	44	123	916	$51.50
57.45	36	503	134	15.29
91.16	28	175	111	12.71
36.00	33	497	132	21.14
86.75	55	202	36	14.97
45.00	70	12	35	4.20
35.08	4	84	47	0.69

146. What is the extent of the British Isles, if Scotland has 30,000 square miles, England 51,200, Wales 7,200, and Ireland 32,500 ?

QUESTIONS.

32. What are like numbers ? 33. What is addition ? 34. What is the sum of two or more numbers ?

35. What is the sign of addition ? 36. The sign of equality ? 37. The dollar sign ?

38. Give a principle of addition ?

39. In expressing currency of the United States in figures, what decimal places may cents occupy ? What decimal place do mills occupy ?

40. How are numbers written for adding ? How are they added ? 41. What is the proof of addition ?

SUBTRACTION.

42. **1.** If John should catch 6 trout and Arthur 4, how many more will John have than Arthur?

2. 4 and how many are 6? How many are 6 less 4?

3. John has been angling 5 hours and Arthur 3. How many more hours has John been angling than Arthur?

4. John sold a trout for 6 cents, and another for 3 cents. How much more did he get for the one than for the other?

5. 5 less 3 are how many? 6 less 3 are how many?

6. Jane had 8 peaches and gave away 4. How many had she left?

7. 9 books less 6 books are how many books?

8. 8 less 3 are how many? 9 less 6 are how many?

9. If you take $4 from $7, how many dollars will be left?

10. Upon a tree there were 8 birds, but 5 have flown away. How many remain? 5 and how many are 8?

11. How many are 7 less 4? 8 less 5?

12. A man earned in a week $9, and spent for board 6. How many dollars had he then left?

13. What is a unit of $9? A unit of $6? Why then are $9 and $6 like numbers? $6 and how many dollars are $9?

43. **Subtraction** is taking one of two like numbers from the other.

44. The **Subtrahend** is the number subtracted.

45. The **Minuend** is the number from which the subtrahend is taken.

46. The **Difference** or **Remainder** is the result of a subtraction.

47. The **Sign of Subtraction** is —. Its name is *minus*, which means *less*.

Thus, $8 - 6 = 2$ is read 8 less 6 are 2.

Copy and complete the following:

SUBTRACTION TABLES.

14.

1 − 1 =
4 − 1 =
3 − 1 =
2 − 1 =
7 − 1 =
6 − 1 =
5 − 1 =
8 − 1 =
9 − 1 =

15.

2 − 2 =
6 − 2 =
4 − 2 =
3 − 2 =
5 − 2 =
9 − 2 =
8 − 2 =
7 − 2 =
11 − 2 =

16.

4 − 3 =
3 − 3 =
6 − 3 =
5 − 3 =
8 − 3 =
7 − 3 =
9 − 3 =
11 − 3 =
10 − 3 =

17.

7 − 4 =
6 − 4 =
4 − 4 =
5 − 4 =
9 − 4 =
8 − 4 =
12 − 4 =
10 − 4 =
13 − 4 =

18.

6 − 5 =
8 − 5 =
5 − 5 =
7 − 5 =
9 − 5 =
12 − 5 =
10 − 5 =
11 − 5 =
14 − 5 =

19.

8 − 6 =
7 − 6 =
6 − 6 =
9 − 6 =
11 − 6 =
13 − 6 =
10 − 6 =
12 − 6 =
15 − 6 =

20.

9 − 7 =
8 − 7 =
7 − 7 =
12 − 7 =
10 − 7 =
11 − 7 =
14 − 7 =
15 − 7 =
13 − 7 =

21.

9 − 8 =
8 − 8 =
13 − 8 =
10 − 8 =
12 − 8 =
11 − 8 =
15 − 8 =
17 − 8 =
16 − 8 =

22.

9 − 9 =
13 − 9 =
12 − 9 =
11 − 9 =
14 − 9 =
10 − 9 =
15 − 9 =
17 − 9 =
16 − 9 =

23. $6 + 8 - 5 = ?$ $7 - 5 + 9 = ?$ $9 + 3 - 7 = ?$

24. $18 + 3 - 7 = ?$ $9 - 6 + 10 = ?$ $11 + 5 - 9 = ?$

25. $8 + 7 - 9 + 3 - 7 = ?$ $11 + 2 - 6 + 5 - 2 = ?$

26. $12 - 5 + 7 - 3 = ?$ $15 + 5 - 9 - 3 + 8 = ?$

27. Subtract 9 from 13. 2 from 11. 7 from 12.

28. Subtract 6 from 8. 8 from 9. 4 from 4.

29. Subtract 1 from 7. 5 from 17. 9 from 16.

30. Subtract 7 from 15. 9 from 18. 4 from 13.

31. Subtract by 2's from 24 back to 0.

Solution. — 22, 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0.

Subtract

32. By 3's from 27 to 6.
33. By 4's from 32 to 4.
34. By 5's from 45 to 10.
35. By 6's from 54 to 12.
36. By 7's from 49 to 21.
37. By 7's from 63 to 42.
38. By 6's from 84 to 60.
39. By 8's from 96 to 72.
40. By 8's from 72 to 40.
41. By 9's from 81 to 45.

48. A Principle of Subtraction.

The sum of the subtrahend and the difference is equal to the minuend.

WRITTEN EXERCISES.

42. Subtract 325 from 958.

Minuend,	958
Subtrahend,	325
Difference,	633
Proof,	958

Solution. — For convenience we write the subtrahend under the minuend, so that figures of the same order are in the same column.

5 ones from 8 ones leave 3 ones, which we write beneath in ones'

place; 2 tens from 5 tens leave 3 tens, which we write underneath in tens' place; and 3 hundreds from 9 hundreds leave 6 hundreds, which we write underneath in hundreds' place.

The difference is 6 hundreds 3 tens 3 ones, or 633.

To test or prove the work, we add the subtrahend and the difference together, and the sum equals the minuend (Art. 48).

	43.	**44.**	**45.**	**46.**
From	97	353	675	563
subtract	46	132	63	341

	47.	**48.**	**49.**	**50.**
From	89	465	763	867
subtract	57	213	551	615

51. A farmer raised 535 bushels of corn, and has sold 225 bushels. How much has he left?

52. How many dollars are $939 less $535?

ORAL EXERCISES.

53. Mary had 8 books and gave her sister 6 of them. How many had she left?

54. If a boy has 13 marbles, and gives away 7 of them, how many has he left?

55. Bought a watch for $10 and sold it for $17. How much was gained?

56. From a brood of 15 chickens the hawks have taken 6. How many remain?

57. If I go a-shopping with 18 cents, and spend 8 cents, how much have I left?

58. Sold goods for $21, which was $5 more than they cost me. What did they cost me?

59. John had 11 cents and paid 5 cents for a pencil. How many had he left?

60. What is 13 – 4? 23 – 4? 33 – 4? 43 – 4? 53 – 4?

61. Sold a knife for 31 cents, which was 6 cents more than it cost me. What did it cost me?

62. Count back from 30 to 0 by subtracting 3's. From 50 to 0 by subtracting 5's?

63. Ellen is 19 years old and her sister is 11. What is the difference in their ages?

64. Henry has 13 apples and Ellen 9. How many more has Henry than Ellen?

65. Count back by 6's from 72 to 6? From 96 to 72?

66. How many will remain if 9 are taken from 19? 11 from 20? 7 from 31?

67. Arthur has 8 cents. How many more cents does he need to make 19 cents?

68. If you have 6 peaches, how many more must you have to make 15 peaches?

69. What is 24 – 7? 35 – 6? 47 – 9?

WRITTEN EXERCISES.

70. From 652 take 423.

Minuend,	652
Subtrahend,	423
Difference,	229
Proof,	652

Solution. — 3 ones cannot be taken from 2 ones; we therefore take 1 ten, or 10 ones, from the 5 tens of the minuend, leaving 4 tens; and adding the 10 ones to the 2 ones we have 12 ones; 3 ones from 12 ones leave 9 ones, which we write beneath in the place of ones.

2 tens from 4 tens leave 2 tens, which we write underneath in the place of tens.

4 hundreds from 6 hundreds leave 2 hundreds, which we write underneath in the place of hundreds.

The difference is 6 hundreds 5 tens 2 ones, or 652. This we prove, by adding the subtrahend and difference (Art. 48).

	71.	**72.**	**73.**	**74.**
From	466	673	526	926
subtract	270	564	213	103

	75.	**76.**	**77.**	**78.**
From	791	926	712	823
subtract	63	19	102	16

	79.	**80.**	**81.**	**82.**
From	823	612	976	312
take	310	13	575	189

	83.	**84.**	**85.**	**86.**
From	433	964	876	847
take	244	109	445	206

ORAL EXERCISES.

87. How many does 6 from 13 leave? 6 from 23? 6 from 43? 6 from 73? 6 from 93?

88. How many does 5 from 12 leave? 5 from 32? 5 from 22? 5 from 42? 5 from 72?

89. How many does 7 from 11 leave? 7 from 21? 7 from 41? 7 from 64? 7 from 84?

90. How many does 8 from 16 leave? 8 from 36? 8 from 76? 8 from 96? 8 from 15? 8 from 55?

91. How many does 9 from 17 leave? 9 from 27? 9 from 37? 9 from 67? 9 from 77? 9 from 88?

92. How many does 3 from 10 leave? 3 from 40? 3 from 60? 3 from 70? 3 from 90?

93. 23 from 47 leaves how many?

94. 17 from 28 leaves how many? 19 from 39?

95. 25 from 46 leaves how many? 35 from 55?

96. There are 21 pupils in one school-room, and 33 in another. How many less are there in the one than in the other?

97. 58 chests of tea less 34 chests, are how many? 34 and how many are 58?

WRITTEN EXERCISES.

98. How many dollars are $68.08 less $45.28?

Minuend,	$68.08
Subtrahend,	45.50
Difference,	$22.58
Proof,	$68.08

Solution. — Here the 5 tenths of the subtrahend cannot be taken from the 0 tenth of the minuend; we therefore take 1 one, or 10 tenths, from the 8 ones of the minuend, leaving 7 ones; 5 tenths from the 10 tenths leave 5 tenths, which we write for the tenths in the difference; subtracting then the ones and tens, we have as the difference $22.58.

	99.	**100.**	**101.**	**102.**
From	$32.28	66.60	$47.00	3.456
take	20.37	56.15	3.50	1.531

	103.	**104.**	**105.**	**106.**
From	4006	2610	1007	4708
take	340	1102	19	80

	107.	**108.**	**109.**	**110.**
From	5987	4062	7910	8008
take	890	1051	7820	909

49. Rule for Subtraction.

Write the subtrahend under the minuend, placing units of the same order in the same column.

Begin with the units of the lowest order to subtract, and proceed to the highest, writing the result beneath.

If any order of the minuend has fewer units than the same order of the subtrahend, increase its units by ten, and subtract. Then consider the units of the next minuend order one less, and proceed as before.

50. Proof. — Add the subtrahend and the difference; their sum should equal the minuend (Art. 48).

111. 463 tons less 309 tons = how many tons?

112. 4612 rods − 2004 rods = how many rods?

113. 31020 − 16101 = how many?

114. 20000 − 1010 = how many?

115. 111111 − 11112 = how many?

116. Subtract 635 from 34621.

117. Subtract 31002 from 31201.

118. Take 416 from 1928.

119. Take 313142 from 404030.

120. I had $1630.50 in the bank, but drew out $625.25. What have I remaining in the bank?

121. Paid $225.75 for a horse, and $125.85 for a buggy. How much more did the horse cost than the buggy?

122. How old would a man be in 1881 who was born in 1819?

123. How much greater is 16.357 than 9.943?

124. The Pilgrims landed at Plymouth in 1620. How many years was this before the Declaration of Independence was adopted, in 1776?

125. John Low bought a farm for $3000, and has paid for it within $709. How much has he paid already?

126. Maryland has 11124 square miles, and Delaware 2120 square miles. How many more square miles has the one than the other?

127. A farmer raised 5723 bushels of corn, and has sold 3846 bushels. How much remains unsold?

128. From ten thousand ten, take one thousand one hundred one.

129. From seventeen thousand five hundred ninety, take twelve thousand nine hundred nine.

130. The smaller of two numbers is 61,342, and the larger 1,000,300. What is their difference?

131. The salary of the President in four years amounts to $200,000; if his expenses in that time are $63,575, how much of his salary will be left?

132. A trader bought 30 casks of cider, containing 1,890 gallons, and sold 23 casks, containing 1,449 gallons. How many casks, and how many gallons, were left?

QUESTIONS.

43. What is subtraction? 44. The subtrahend? 45. The minuend? 46. The difference?

47. What is the sign of subtraction? What does it mean?

48. What is a principle of subtraction?

49. How are the numbers written for subtraction? How do you subtract? What is done if any order of the minuend has a less number of units than the same order of the subtrahend? What is the proof?

REVIEW.

ORAL EXERCISES.

51. 1. John has 17 apples and James 8. How many more than both of them has Andrew, who has 30?

2. Peter hired a horse to go 15 miles, but he went 6 miles farther. When 10 miles on his way back, how many miles had he traveled?

3. How many are 25 + 11 + 12?

4. How many are 31 + 19, less 20?

5. From $28 take the sum of $8 and $11.

6. Add 13 and 21 and take 15 from their sum.

7. The sum of John's money and Arthur's is $37. John has $29; how much has Arthur?

8. The difference in the ages of two men is 7 years, and the younger is 23 years old. How much older is the other?

9. I had 2 twenty-dollar bills and 1 ten-dollar bill, and $1.50 in change, but have spent from it $5.50 for a hat, $5 for a pair of boots, and $30 for a suit of clothes. How much money have I left?

10. How much must be added to 35 − 16 to make 30?

11. Susan has 36 books, which is 32 less than Mary has. How many books has Mary?

12. Bought flour for $8.50, sugar for $3.50 and other articles amounting to $21, and gave in payment a twenty-dollar bill, a ten-dollar bill, and a five-dollar bill. How much change should I receive?

WRITTEN EXERCISES.

13. The difference is 3918 and the subtrahend 3614. What is the minuend?

14. Columbus discovered America in 1492. How long was it from that time to 1880?

15. The difference is 9088, and the minuend 17630. What is the subtrahend?

16. A man having $78.39, paid away at one time $3.17, and at another $21.63. What did he then have?

17. Bought flour to the amount of $256, cloth to the amount of $317, and paid down $373. How much remained to be paid?

18. Henry Smith went out with $435 to settle accounts. He paid $115 to one man, received $119 from another, and paid $316 to another. How much had he left?

19. A boy had 216 apples; he sold 34 of them one day, 72 another day, and afterwards 29; how many had he left?

20. Bought a house for $3165, laid out for repairs on it $593, and sold it for $3500. How much was lost by the bargain?

21. Sold cloth for $10, sugar $17, pork $43, beef $19, and fish $13; and in payment received a check for $75, and a 50-dollar note. How much change should be paid back?

22. A farmer buys a yoke of oxen for $163. He exchanges them for others, paying $27, and then sells for $200. Did he gain or lose, and how much?

23. Dana estimates that there are workable coal beds in the United States to the extent of 130000 square miles; and in Great Britain and Ireland of 12000, in Spain of 4000, in France of 2000, and in Belgium of 518. How much do those of the United States exceed those of the other countries named?

REVIEW QUESTIONS.

2. What is a number? 32. What are like numbers?

24. What is notation? 25. What is numeration?

21. What is a group? What is used to mark off groups? 23. What is used to distinguish the order of ones?

22. What are decimals? 29. How are decimals read?

33. What is addition? 43. What is subtraction? 41. What is the proof in addition? 50. What is the proof in subtraction?

MULTIPLICATION.

52. **1.** Kate has 5 letters and Emma has 3 times as many. How many has Emma?

2. Charles gave 3 apples to each of his 6 schoolmates. How many did he give them in all?

3. How many are 2 times 5? 3 times 6?

4. If a boy earns 4 dollars in a week, how much will he earn in 7 weeks?

5. If you obtain each day 7 merits, how many will you receive in 6 days?

6. How many are 8 times 7? 6 times 7?

7. What cost 3 quarts of berries at 8 cents a quart?

8. How much must be paid for 8 quarts of milk at 6 cents a quart?

9. What will 5 pounds of rice cost at 9 cents a pound?

10. How many are 3 times 8? 6 times 8? 5 times 9?

11. If a horse can trot 7 miles in an hour, how many miles can he trot in 4 hours?

12. If one boat can carry 9 persons, how many persons can 6 boats carry?

13. How many are 4 times 7? 6 times 9? 7 times 4? 9 times 6?

53. **Multiplication** is taking a number as many times as there are ones in another.

54. The **Multiplicand** is the number to be taken or multiplied.

55. The **Multiplier** is the number that shows how many times the multiplicand is to be taken.

56. The **Product** is the result of a multiplication.

The *factors* of a product, or of a number, are the numbers multiplied together to produce it.

57. The **Sign of Multiplication** is ×. It means *multiplied by*, or *times.*

Thus, 7×5 may be read seven multiplied by five, or seven times five.

Copy and recite the following

MULTIPLICATION TABLES.

14.	**15.**	**16.**
$2 \times 1 = 2$	$3 \times 1 = 3$	$4 \times 1 = 4$
$2 \times 2 = 4$	$3 \times 2 = 6$	$4 \times 2 = 8$
$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$
$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$
$2 \times 5 = 10$	$3 \times 5 = 15$	$4 \times 5 = 20$
$2 \times 6 = 12$	$3 \times 6 = 18$	$4 \times 6 = 24$
$2 \times 7 = 14$	$3 \times 7 = 21$	$4 \times 7 = 28$
$2 \times 8 = 16$	$3 \times 8 = 24$	$4 \times 8 = 32$
$2 \times 9 = 18$	$3 \times 9 = 27$	$4 \times 9 = 36$
$2 \times 10 = 20$	$3 \times 10 = 30$	$4 \times 10 = 40$

17.

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 3 = 15$
$5 \times 4 = 20$
$5 \times 5 = 25$
$5 \times 6 = 30$
$5 \times 7 = 35$
$5 \times 8 = 40$
$5 \times 9 = 45$
$5 \times 10 = 50$

18.

$6 \times 1 = 6$
$6 \times 2 = 12$
$6 \times 3 = 18$
$6 \times 4 = 24$
$6 \times 5 = 30$
$6 \times 6 = 36$
$6 \times 7 = 42$
$6 \times 8 = 48$
$6 \times 9 = 54$
$6 \times 10 = 60$

19.

$7 \times 1 = 7$
$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
$7 \times 5 = 35$
$7 \times 6 = 42$
$7 \times 7 = 49$
$7 \times 8 = 56$
$7 \times 9 = 63$
$7 \times 10 = 70$

20.

$8 \times 1 = 8$
$8 \times 2 = 16$
$8 \times 3 = 24$
$8 \times 4 = 32$
$8 \times 5 = 40$
$8 \times 6 = 48$
$8 \times 7 = 56$
$8 \times 8 = 64$
$8 \times 9 = 72$
$8 \times 10 = 80$

21.

$9 \times 1 = 9$
$9 \times 2 = 18$
$9 \times 3 = 27$
$9 \times 4 = 36$
$9 \times 5 = 45$
$9 \times 6 = 54$
$9 \times 7 = 63$
$9 \times 8 = 72$
$9 \times 9 = 81$
$9 \times 10 = 90$

22.

$10 \times 1 = 10$
$10 \times 2 = 20$
$10 \times 3 = 30$
$10 \times 4 = 40$
$10 \times 5 = 50$
$10 \times 6 = 60$
$10 \times 7 = 70$
$10 \times 8 = 80$
$10 \times 9 = 90$
$10 \times 10 = 100$

23. $3 \times 2, + 9 = ?$ $9 \times 6, - 8 = ?$ $72 = 6 \times ?$

Note. — The comma is here used to show that the operation indicated by each sign is to be performed on the result of the preceding operation.

24. $4 + 5, \times 9 = ?$ $17 - 8, \times 7 = ?$ $132 = 11 \times ?$

25. $29 - 8, - 7, - 6, \times 8, + 9 = ?$ $2 \times 2 \times 2 \times 7 = ?$

26. $6 \times 7, - 9, + 7, + 8, - 5 = ?$ $67 + 5, - 9 = 7 \times ?$

27. 6 × 9, − 3, − 7, − 8, − 10, − 5, + 8, + 9, + 2, + 5, + 3 = 6 × ?

28. Take 6 × 3 from 7 × 4 and add the result to 5 × 7.

29. Give the products of 7 times 8, 9, 12, 5, 7, 11, 3.

30. Count by 6's from 6 to 72.

Solution.—6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72.

Give the product

31. Of 3's from 3 times 1 to 3 times 9.

32. Of 4's from 4 times 0 to 4 times 9.

33. Of 5's from 5 times 1 to 5 times 10.

34. Of 6's from 6 times 0 to 6 times 8.

35. Of 7's from 7 times 1 to 7 times 9.

36. Of 8's from 8 times 2 to 8 times 10.

37. Of 9's from 9 times 1 to 9 times 10.

38. Of 10's from 10 times 1 to 10 times 10.

58. Principles of Multiplication.

1. *The product is the same in whatever order the factors may be taken.*

2. *Multiplicand and product are always like numbers.*

WRITTEN EXERCISES.

39. Multiply 75 by 7.

Multiplicand,	75
Multiplier,	7
Product,	525

Solution.—We write the multiplier, 7 ones, under the ones' figure of the multiplicand.

Beginning with the ones we multiply, 7 times 5 ones are 35 ones, or 3 tens 5 ones. We write the 5 ones beneath in the place of ones, and reserve the 3 tens.

7 times 7 tens are 49 tens, and 49 tens + 3 tens reserved are 52 tens, or 5 hundreds 2 tens. We write beneath the 2 tens in tens' place, and the 5 hundreds in hundreds' place.

The product is 5 hundreds 2 tens 5 ones, 525.

To test or prove the work, we carefully perform it a second time.

	40.	**41.**	**42.**	**43.**
Multiply	48	63	71	82
by	5	4	7	6

	44.	**45.**	**46.**	**47.**
Multiply	82	61	45	98
by	3	8	9	7

	48.	**49.**	**50.**	**51.**
Multiply	61	23	892	904
by	5	8	9	4

	52.	**53.**	**54.**	**55.**
Multiply	$1.61	$8.06	$1.39	$6.78
by	3	6	7	8

56. What is the product of 225 by 7?

57. How much must be paid for 8 barrels of flour at $9.75 per barrel?

58. How many acres in 3 farms of 340 acres each?

59. How many pounds will 8 barrels of flour weigh, the weight of a barrel being 196 pounds?

ORAL EXERCISES.

60. If a man travels 5 miles in one hour, how far will he travel in 7 hours?

61. What will 9 writing-books cost at 12 cents each?

62. How much must be paid for 4 tons of coal at 7 dollars a ton?

63. How many birds are 6 times 6 birds?

64. If 4 men build a fence in 9 days, how many men can build it in one day?

65. What will 12 barrels of flour cost at 8 dollars a barrel?

66. How many hills are there in 12 rows of 12 hills each?

67. What cost 11 pencils at 10 cents each?

68. By reading 9 pages a day, how many pages can you read in 9 days?

69. If you get 9 merits each day, how many merits can you get in 8 days?

70. What cost 7 quarts of cherries at 7 cents a quart?

71. How much must be paid for 11 tons of coal at 5 dollars a ton?

72. If a boat sails 9 miles an hour, how far will it sail in 12 hours?

73. When flour is 8 dollars a barrel, how much must be paid for 6 barrels?

74. John is 8 years old and his father is 4 times as old. How old is his father?

75. How many are 4 times 24?

Solution. — 24 is 2 tens 4 ones; 4 times 2 tens are 8 tens, or 80 ones; and 4 times 4 ones are 16 ones, which added to the 80 ones make 96. 4 times 24 are 96.

76. How many are 3 times 16? 3 times 35?

77. How many are 5 times 18? 4 times 27?

78. How many are 6 times 31? 7 times 22?

79. At $4 a yard how much must be paid for 40 yards of velvet?

WRITTEN EXERCISES.

80. Multiply 13 by 10; 13 by 100.

Multiplicand,	13	13
Multiplier,	10	100
Product, .	130	1300

Solution. — 10 thirteens is the same as 13 tens (Art. 58), or 130. Also, 100 thirteens is the same as 13 hundreds, or 1300.

That is,

59. When the multiplier is 10, 100, etc.,

The product may be obtained by simply annexing to the multiplicand as many zeros as there are in the multiplier, and moving the decimal point an equal number of places to the right.

81. 68 × 100.

82. $1.50 × 10.

83. $1.65 × 100.

84. 6.85 × 10.

85. 56 × 100.

86. 156 × 1000.

87. Multiply 75 by 4 and by 10, and add the products.

88. Multiply 64 by 2 and the product by 10.

89. Multiply 73 by 30.

Multiplicand,	73
Multiplier,	30
Product,	2190

Solution. — 30 is 10 times 3. 30 times 73 is the same as 10 times 3 times 73. 3 times 73 is 219, and 10 times 3 times 73 is 10 times 219, or 2190.

Multiply

90. 95 by 40.

91. 56 by 70.

92. 46 by 80.

93. $6.51 by 90.

94. Multiply 63 by 5 and by 30, and add the products.

95. Multiply 183 by 35.

Multiplicand,	183
Multiplier,	35
Partial	915
Products,	549
Product,	6405

Solution. — We write the multiplier under the multiplicand, ones under ones, and tens under tens.

Multiplying by the 5 ones we have 915 as a partial product.

Multiplying by the 3 tens we have 549 tens as a partial product, which we write so that the 9 shall come in the same column as the figure of the multiplier used to produce it.

Adding these partial products we have as the entire product 6405.

Multiply

96. 365 by 25.

97. 981 by 44.

98. 617 by 121.

99. 736 by 206.

Multiplicand,	736
Multiplier,	206
Partial	4416
Products,	1472
Product,	151616

Solution. — Multiplying by the 6 ones, we have as a partial product 4416.

There being 0 tens in the multiplier we pass to the hundreds of the multiplier.

Multiplying by the 2 hundreds, we have as a partial product 1472 hundreds.

Adding the partial products we have as the entire product 151616.

100. Multiply 162 by 103.

101. Multiply 802 by 401.

102. Multiply 534 by 242, and prove the work.

$$\begin{array}{rcr}
534 & \quad & 242 \\
242 & & 534 \\
\hline
1068 & & 968 \\
2136 & & 726 \\
1068 & & 1210 \\
\hline
129228 & & 129228
\end{array}$$

Solution. — Multiplying by the ones, tens, and hundreds of the multiplier, and adding the partial products, we have as the entire product 129228.

To prove the work, since 242 times 534 is the same as 534 times 242, we multiply 242 by 534, and have, as before, 129228 (Art. 58).

Multiply and prove the work —

103. 375 by 412.

104. 260 by 95.

105. 529 by 207.

106. 8.09 by 223.

107. 9.16 by 250.

108. 12355 × 678.

109. 43705 × 905.

110. 896 × itself.

111. 34760 × 809.

112. 8295 × 894.

60. Rule for Multiplication.

Write the multiplier under the multiplicand, drawing a line beneath.

Beginning at the right multiply each order of the multiplicand by each order of the multiplier successively, plac-

ing the right-hand figure of each partial product under the figure of the multiplier used to produce it.

Add the partial products, and from the right of the result point off as many decimal places as are found in both factors.

61. PROOF. — Take the multiplicand for the multiplier, and the multiplier for the multiplicand, and find their product.

113. How much will a farm of 64 acres cost at $110 an acre?

114. If a ship sails 192 miles a day, how many miles will it sail in 56 days?

115. What will 49 yards of cloth cost at $0.37 a yard?

116. If a coach wheel turns round 346 times in going a mile, how many times will it turn round in going 95 miles?

117. What will 145 tons of hay cost at $20 a ton?

118. 68920 × 100 = ?

119. 3161 × 204 = ?

120. 7812 × 905 = ?

121. 31.07 × 807 = ?

122. 19006 × 349 = ?

123. 4671 × 821 = ?

124. 12.45 × 555 = ?

125. 340.92 × 635 = ?

126. What will 320 barrels of flour cost at $9.50 a barrel?

127. What is the value of 130 bonds at $113 each?

128. What will 120 yards of cloth cost at $3.50 a yard?

129. 5345 × 1000 = ?   3100 × 990 = ?

130. 1504 × 316 = ?   7755 × 480 = ?

131. 4309 × 560 = ?   8080 × 703 = ?

132. 9230 × 444 = ?   9103 × 809 = ?

133. If the average number of beats of the heart in childhood is 105 in a minute, what will be the number per hour, or 60 minutes?

134. In one square mile there are 640 acres, and in Delaware there are 2110 square miles. How many acres in the State?

135. If the sailing distance from Liverpool to New York is 3050 miles, how far will a vessel sail in making 40 complete trips?

136. If the earth in its orbit around the sun moves at the rate of 68000 miles an hour, how far does it move in one day, or 24 hours?

137. Light travels 186000 miles in 1 second; how far will it travel in 8 minutes of 60 seconds each?

QUESTIONS.

53. What is multiplication? 54. What is the multiplicand? 55. The multiplier? 56. The product?

57. What does the sign of multiplication mean? 58. Give two principles of multiplication.

60. How do you write the multiplier and multiplicand for multiplying? If the multiplier consists of but one order of units how do you multiply? If of more than one order of units? 61. What is the proof?

ORAL EXERCISES.

62. 1. In one inclosure a farmer has 6 sheep, in another 10, and in a third twice as many as in the first. How many sheep are there in the three inclosures?

2. How many are 6 + 10 + 2 × 6?

3. What is the product of 12 × 9?

4. What number added to 6 × 6 will make 40?

5. What is three times the difference between 4 × 5 and 6 × 5?

6. What is the value of 7 + 8 − 11? Of 16 + 3 − 9?

7. How much greater is 9 + 7 than 9 − 7?

8. The minuend is 23, and the subtrahend 12. What is the difference?

9. In my purse there was $16; I paid away $11, and returned $9. How much was there then in the purse?

10. A man earned $8 a week for 5 weeks, and paid for board $15. How many dollars will be left?

11. Sold a cow for $50, which was $12 less than cost? What was the cost?

12. Bought 20 books at one time, and 26 at another, and sold 30. How many had I then left?

13. Bought candy for 9 cents, fruit for 11 cents, and cake for 12 cents, and gave in payment two 25 cent pieces. What change should I receive back?

14. Twenty-nine, and five, and six, and five times four are how many?

15. Seven times eight, less six, plus eight are how many?

16. John has 8 apples, and Arthur has 9 times as many, less 12. How many has Arthur?

17. A woman went to a store with 75 cents. She bought 3 pounds of sugar at 10 cents a pound and 4 pounds of rice at 9 cents a pound. How much money had she left?

18. A farmer had in his garden 15 trees; he set out 3 times as many more. If 13 should die how many would there remain alive?

19. 15, plus 3 times 15, less 13, are how many?

20. Two men start from the same place; one goes at the rate of 6 miles an hour, and the other at the rate of 5 miles an hour. How many miles more than 48 are they apart at the end of 5 hours.

WRITTEN EXERCISES.

21. Find the product of 2843 and 927.

22. 34768 + 9436 subtracted from 84 times 98720 = ?

23. Find the sum of the numbers in exercises 21 and 22 ?

24. Multiply 846 by itself, and take from the product 2516.

25. The subtrahend is 84963 and the minuend 100000. Find the difference.

26. One factor is 849, the other 9862; find the product.

27. Bought 20 tons of coal at $7.25 a ton and 8 cords of wood at $5.13 a cord. What was the cost of the whole ?

Solution.

20 tons cost 20 × $7.25, or	$145.00
8 cords cost 8 × $5.13, or	41.04
The wood and the coal together cost . .	$186.04

28. If 37 trees yield 13 bushels of apples each, and 41 others yield 11 bushels each, how many bushels do they all yield ?

29. Bought 30 yards of cloth at $3.25 a yard, 7 bales of batting at $13 a bale, and other goods amounting to $21.50. What was the cost of the whole ?

30. Sold 325 barrels of apples at $3 a barrel, and received $650 in cash, and a note for the balance. For what sum was the note ?

31. A farmer bought 32 sheep at $6 each, 31 cows at $45 each, and 3 horses at $162 each. He sold the whole for $2500; how much did he gain?

32. Two men start from the same place and travel in opposite directions, the one at the rate of 32 miles a day, and the other at the rate of 34 miles a day. How far apart will they be at the end of 6 days?

33. Sold 560 tons of coal at $9 a ton, and received in payment 875 bushels of wheat at $2 per bushel, and the balance in money. How much money did I receive?

34. A merchant had in a bank $8000, drew out $7563, and afterwards put in $1550; how much had he then in the bank?

35. Bought 7 rolls of carpeting, each containing 27 yards, and 4 rolls, each containing 18 yards. How many yards were bought in all?

36. Sold 476 barrels of oil at $4.50 a barrel, and received $950 in cash, and a horse for the balance. What was the value of the horse?

37. Arthur has $35 and his brother 4 times as much, lacking $15. How much has his brother?

38. A man having in business $9600, drew out $5367, and afterwards put in 4 times as much as he drew out. How much had he then in the business?

39. Bought one farm of 63 acres at $105 per acre, and another of 240 acres at $95 per acre. What did both farms cost?

40. Sold a farm for $1600.75, 10 acres of land at $65.50 per acre, and a horse for $165. How much was obtained for all?

41. Bought a dictionary for $6.50, a set of histories for $22, and a library for $137.75, and gave in payment a check for $63, and three 50-dollar bills. How much should be received back?

42. Bought 31 acres of land at $145.50 per acre, and gave in payment 21 United States bonds at $111 each, twenty 100-dollar bills, and my note for the balance. For what sum did I give my note?

43. My money is $60.80, and Smith's is $28 more than 5 times as much. How much has Smith?

44. Two steamers start at the same time to meet each other, the one from Boston and the other from Liverpool, which places are 2883 miles apart. Provided that one sails at the rate of 192 miles a day, and the other at the rate of 168 miles a day, how many miles will they be from each other at the end of 5 days?

REVIEW QUESTIONS.

34. What is the answer called in addition? 46. In subtraction? 56. In multiplication?

35. What does the addition sign mean? 47. The subtraction sign? 57. The multiplication sign?

28. What is a principle of notation? 38. A principle of addition? 48. A principle of subtraction? 58. A principle of multiplication?

41. What is the proof in addition? 50. In subtraction? 61. In multiplication?

DIVISION.

63. 1. John has 8 apples; how many times 2 apples has he?

2. James and his brother together have 10 apples; if they share them equally, how many will each have?

3. How many times 2 in 8? In 10? In 16?

4. If 3 boys have 9 marbles together, and should share them equally, how many would each boy have?

5. Into how many equal groups of 3 marbles each, can 9 marbles be separated?

6. What is one of the 3 equal parts of 9? One of the 4 equal parts of 8?

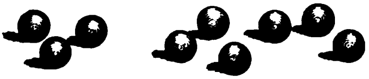
7. If 4 barrels make a dray-load, how many dray-loads will 12 barrels make?

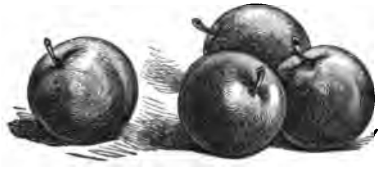
8. How many times 4 barrels in 12 barrels? 3 barrels in 12 barrels?

64. One of the *two* equal parts of a number is called **one half.**

Thus, one half of 2 apples is 1 apple; of 4 apples is 2 apples.

One of the *three* equal parts of a number is called **one third.**





Thus, one third of 9 marbles is 3 marbles.

One of the *four* equal parts of a number is called **one fourth.**

Thus, one fourth of 4 apples is one apple; one fourth of 8 apples is 2 apples.

One of the *five* equal parts of a number is called **one fifth.**

One of the *six* equal parts of a number is called **one sixth,** etc.

65. **Division** is finding how many times one number is contained in another; or finding one of the equal parts of a number.

66. The **Dividend** is the number to be divided.

67. The **Divisor** is the number by which we divide.

68. The **Quotient** is the result of a division.

69. The **Remainder** is the part of the dividend left, when the latter does not contain the divisor an exact number of times.

70. The **Sign of Division,** ÷, or :, means *divided by.*

Thus, 16 ÷ 8, or 16 : 8, is read sixteen divided by eight.

Division is also indicated by writing the divisor at the left and the dividend at the right of a curve.

Thus, 6) 48 is read 6 in 48.

Copy and complete the following

DIVISION TABLES.

9.

$2 \div 2 =$
$8 \div 2 =$
$6 \div 2 =$
$4 \div 2 =$
$10 \div 2 =$
$12 \div 2 =$
$18 \div 2 =$
$16 \div 2 =$
$14 \div 2 =$
$20 \div 2 =$

10.

$3 \div 3 =$
$6 \div 3 =$
$15 \div 3 =$
$12 \div 3 =$
$9 \div 3 =$
$18 \div 3 =$
$27 \div 3 =$
$24 \div 3 =$
$30 \div 3 =$
$21 \div 3 =$

11.

$8 \div 4 =$
$4 \div 4 =$
$12 \div 4 =$
$28 \div 4 =$
$20 \div 4 =$
$24 \div 4 =$
$16 \div 4 =$
$32 \div 4 =$
$40 \div 4 =$
$36 \div 4 =$

12.

$10 \div 5 =$
$5 \div 5 =$
$15 \div 5 =$
$20 \div 5 =$
$35 \div 5 =$
$30 \div 5 =$
$25 \div 5 =$
$50 \div 5 =$
$45 \div 5 =$
$40 \div 5 =$

13.

$6 \div 6 =$
$24 \div 6 =$
$18 \div 6 =$
$12 \div 6 =$
$36 \div 6 =$
$30 \div 6 =$
$54 \div 6 =$
$48 \div 6 =$
$42 \div 6 =$
$60 \div 6 =$

14.

$7 \div 7 =$
$14 \div 7 =$
$35 \div 7 =$
$28 \div 7 =$
$21 \div 7 =$
$42 \div 7 =$
$70 \div 7 =$
$56 \div 7 =$
$63 \div 7 =$
$49 \div 7 =$

15.

8 ÷ 8 =
32 ÷ 8 =
24 ÷ 8 =
16 ÷ 8 =
80 ÷ 8 =
48 ÷ 8 =
56 ÷ 8 =
40 ÷ 8 =
72 ÷ 8 =
64 ÷ 8 =

16.

9 ÷ 9 =
27 ÷ 9 =
36 ÷ 9 =
18 ÷ 9 =
45 ÷ 9 =
54 ÷ 9 =
63 ÷ 9 =
90 ÷ 9 =
81 ÷ 9 =
72 ÷ 9 =

17.

20 ÷ 10 =
70 ÷ 10 =
10 ÷ 10 =
40 ÷ 10 =
60 ÷ 10 =
30 ÷ 10 =
90 ÷ 10 =
80 ÷ 10 =
50 ÷ 10 =
100 ÷ 10 =

18. How many times 4 in 12? 5 in 10? 3 in 27? 8 in 64?

19. How many times 2 in 16? 6 in 42? 9 in 18? 3 in 18?

20. How many times 7 in 49? 4 in 16? 5 in 50? 9 in 54?

21. How many times 2 in 8? 6 in 18? 3 in 27? 7 in 56?

22. How many times 2 in 20? 6 in 36? 9 in 81? 3 in 30?

23. How many times 5 in 15? 8 in 24? 9 in 45? 5 in 40?

24. How many times 7 in 63? 8 in 72? 3 in 30? 5 in 35?

25. How many times 7 in 28? 8 in 16? 7 in 14? 9 in 63?

26. How many times 10 in 80? 6 in 54? 4 in 36? 10 in 100?

27. Divide by 2 from 2 in 2 to 2 in 20.

Solution. — 2 in 2, once; 2 in 4, 2 times; 2 in 6, 3 times; 2 in 8, 4 times; 2 in 10, 5 times; 2 in 12, 6 times; 2 in 14, 7 times; 2 in 16, 8 times; 2 in 18, 9 times; 2 in 20, 10 times.

Divide

28. By 3 from 3 in 3 to 3 in 30.
29. By 4 from 4 in 4 to 4 in 40.
30. By 5 from 5 in 5 to 5 in 50.
31. By 6 from 6 in 6 to 6 in 60.
32. By 7 from 7 in 7 to 7 in 70.
33. By 8 from 8 in 8 to 8 in 80.
34. By 9 from 9 in 9 to 9 in 90.
35. By 10 from 10 in 10 to 10 in 100.

71. A Principle of Division.

The dividend is the product of divisor and quotient.

WRITTEN EXERCISES.

36. Divide 858 by 6.

Divisor, 6) 858, Dividend.
143, Quotient.
6
Proof, 858

Solution. — We write the divisor at the left of the dividend with a curve between, and begin at the left to divide.

6 in 8 hundreds, 1 hundred times, and 2 hundreds or 20 tens remain. We write the 1 hundred beneath in hundreds' place, and uniting the 20 tens with the 5 tens we have 25 tens.

6 in 25 tens, 4 times and 1 ten, or 10 ones remain. We write the 1 ten beneath in tens' place, and uniting the 10 ones with the 8 have 18.

6 in 18, 3 times ; we write the 3 underneath in ones' place.

The quotient is 1 hundred 4 tens 3 ones, or 143.

To prove the work, we multiply the quotient by the divisor and have as the product the dividend (Art. 71).

37.	**38.**	**39.**	**40.**
3) 645	5) 740	4) 452	6) 894

41.	**42.**	**43.**	**44.**
3) $4.83	4) $5.56	2) $3.78	7) $8.82
$1.61			

45. A father divided $675 equally among his 5 children. How much did each receive?

46. The distance between two seaports is 927 miles. How many hours will it take a steamer at 9 miles an hour to go that distance?

72. The **Equal Parts** of a number may be expressed in figures by writing the number denoting the name of the parts below a short horizontal line, and the number, denoting how many of these parts are taken or used, above the line. Thus,

1 of 2 equal parts is written $\frac{1}{2}$, read *one half,*
1 of 3 equal parts " $\frac{1}{3}$, " *one third,*
2 of 3 equal parts " $\frac{2}{3}$, " *two thirds,*
1 of 4 equal parts " $\frac{1}{4}$, " *one fourth,*
2 of 4 equal parts " $\frac{2}{4}$, " *two fourths,*
3 of 4 equal parts " $\frac{3}{4}$, " *three fourths,*

and so on.

The number written above the line may be considered as a dividend, the number below the line as a divisor, and the expression will be that of division. Thus,

$\frac{1}{4}$ may be read one divided by four.
$\frac{3}{5}$ may be read three divided by five.

73. An **Integer,** or **Whole Number,** is a collection of ones.

Thus, 2, 16, 347, are each integers.

47. Divide 1703 by 4.

$$\begin{array}{r} 4\underline{)\,1703} \\ 425\frac{3}{4} \\ \underline{4} \\ \text{Proof, } 1703 \end{array}$$

Solution. — Dividing we have a final remainder of 3. The division of this we indicate by writing the divisor 4 under the 3, with a short horizontal line between. The quotient, $425\frac{3}{4}$, is read four hundred twenty-five and three fourths.

To prove the work we multiply the integer of the quotient by the divisor, and adding the remainder, have the dividend.

48. 5) 1805

49. 7) 763

50. 3) 694

51. 9) 1890

52. 4) 8126

53. 7) 3514

54. 8) 3464

55. 3) 1005

56. 6) 2405

57. 7) 805

58. 4) 4248

59. 9) 7200

60. 3) 5180

61. 7) 70851

62. 8) 90376

63. 9) 93081

Divide —

64. 57848 by 8.

65. 3616 by 4.

66. 22275 by 7.

67. 57848 by 9.

68. $65412 \div 3 = ?$

69. $50405 \div 5 = ?$

70. $947 \div 4 = ?$

71. $\frac{54686}{8} = ?$

72. At $6 each, how many sheep can be purchased for $1806?

73. Seven men working together earned $1547. What is each man's share of it?

74. A gentleman divided among his 4 sons $24.56. How much did each receive?

75. If a man can do a piece of work in 365 days, how many days will it take 5 men to do the same?

76. John Stevens sold melons at 9 cents each, to the amount of 2943 cents. How many did he sell?

77. If a prize amounting to $951.63 is divided among 8 men, how much will each receive?

78. At $6 a ton how many tons of coal can be bought for $19201?

ORAL EXERCISES.

79. How many oranges at 5 cents each can be bought for 40 cents?

80. How many pounds of prunes at 12 cents each can be bought for 96 cents?

81. At $7 a ton, how many tons of coal can be bought for $49?

82. At 10 cents each, how many writing-books can be bought for 90 cents?

83. How many pencils at 4 cents each can be bought for 24 cents?

84. How many 6's in 42? 7's in 49? 9's in 72?

85. At 3 cents each, how many lemons can be bought for 27 cents?

86. A factory has 64 windows; how many times 8 windows has it?

87. What part of $10 are $2? What is one fifth of 10?

88. What is one sixth of 72? One seventh of 49?

89. If 90 cents be divided among 10 boys, how many cents will each boy receive? What is one tenth of 100?

90. What is the value of $\frac{42}{6}$? Of $63 \div 7$? Of $\frac{85}{7}$?

91. How many sacks holding 8 quarts each can be filled from 75 quarts, and how many quarts will remain?

92. I have $150; how many coats at $12 each can I buy with the money, and how much will remain?

WRITTEN EXERCISES.

93. Divide 1185 by 12.

$$\begin{array}{rl}
 & 98\tfrac{9}{12}\text{, Quotient.} \\
\text{Divisor, } 12) & 1185\text{, Dividend.} \\
 & \underline{108} \\
 & 105 \\
 & \underline{96} \\
 & 9
\end{array}$$

Solution.—12 is in 118 tens, 9 tens times. We write the 9 tens over the tens of the dividend for the tens of the quotient. 9 tens times 12 = 108 tens, which taken from the 118 tens of the dividend, leaves 10 tens; uniting with these 10 tens the 5 ones of the dividend, we have 105.

12 is in 105, 8 times. We write the 8 for the ones of the quotient. 8 times 12 = 96, which taken from the 105 of the dividend leaves 9, and indicating its division by the divisor, have $\frac{9}{12}$. Writing the $\frac{9}{12}$ as a part of the quotient, we have as the quotient 98$\frac{9}{12}$.

Divide —

94. 1748 by 18.

95. 1621 by 21.

96. 1052 by 15.

97. 2944 by 46.

98. Divide 3045 by 15.

$$\begin{array}{r} 203\frac{1}{15} \\ 15)\overline{3046} \\ \underline{30} \\ 46 \\ \underline{45} \\ 1 \end{array}$$

Solution. — 15 in 30 hundreds, 2 hundred times. We write the 2 hundreds for the hundreds of the quotient. 2 hundreds times 15 = 30 hundreds, which taken from the 30 hundreds of the dividend leaves 0 hundreds.

15 is contained in the 4 tens of the dividend 0 ten times. We write the 0 tens for the tens of the quotient, and uniting with the 4 tens the 6 ones of the dividend, we have 46.

15 is contained in 46, 3 times. We write the 3 for the ones of the quotient. 3 times 15 = 45, which taken from the 46 of the dividend leaves 1, and indicating its division by the divisor, we have $\frac{1}{15}$; writing the $\frac{1}{15}$ as a part of the quotient we have as the result required 203$\frac{1}{15}$.

Divide —

99. 5226 by 13.

100. 1785 by 17.

101. 9125 by 25.

102. 3762 by 24.

103. $27.80 by 16.

104. $96.14 by 19.

74. To divide one number by another.

Rule.

Write the divisor at the left of the dividend.

Find how many times the divisor is contained in the least number of the left-hand orders of the dividend that will contain the divisor, and write the result for the first left-hand figure of the quotient.

Multiply the divisor by this quotient, subtract the product from the part of the dividend used, and to the remainder unite the next order of the dividend.

Divide and proceed as before, until all of the dividend has been divided.

If there is a final remainder, write it with the divisor beneath, as a part of the quotient.

NOTE. — The division is called *Short Division* when only the divisor, dividend, and quotient are written; and *Long Division* when each process of the solution is written.

In short division the quotient is usually written below the dividend, and in long division over, or at the right of, the dividend.

When the divisor is an integer, and the quotient is under or over the dividend, the decimal point in the quotient must be placed immediately under or over the point in the dividend.

75. Proof. Multiply the quotient by the divisor, and to the product add the remainder if any.

Divide —

105. 6780 by 15.
106. 3762 by 24.
107. 6318 by 27.
108. 9990 by 54.
109. 78.93 by 13.
110. 91.75 by 65.
111. $415.50 by 65.
112. $657.05 by 73.

113. 45678 by 16.

114. 34331 by 11.

115. $343.31 by 11.

116. $1516.16 by 206.

117. If a ship sails 3672 miles in 24 days, at what rate is that per day?

118. If $20304 is paid for 16 acres of land, what is the price per acre?

119. If I divide $660 equally among 15 men, how much will each receive?

120. In how many hours will a railroad car run 1116 miles at the rate of 21 miles an hour?

121. If I divide $19.32 equally among 12 boys, how many cents will each receive?

122. What is the value of 42024 ÷ 103?

123. What is the value of 208126 ÷ 345?

124. In how many months, at $48 per month, can a man pay for a farm costing $6972?

125. If 21 bushels of wheat weigh 1260 pounds, what is the weight of one bushel?

126. If 11 acres yield 746 bushels of corn, what is the yield per acre?

127. If the sailing distance from New York to San Francisco, by way of Cape Horn, is 18,850 miles, how long will it take a vessel to sail that distance at the rate of 104 miles a day?

128. The salary of the President of the United States is $50000 a year; how much is that a day, there being 365 days in a year?

ORAL EXERCISES.

129. At $10 a ton how many tons of hay can be bought for $100? How many 10's in 100?

130. At $100 each, how many horses can be bought for $1000? How many 100's in 1000?

131. How many 10's in 20? In 60? In 120?

132. How many hundreds in 600? In 1100? In 4600?

133. In 25 how many 10's and what over? In 136 how many tens and what over?

134. In 675 how many hundreds and what over? In 1475 how many hundreds and what over?

135. How many times 30 in 120? How many times $30 \div 10$ in $120 \div 10$?

76. Principle.

Dividing both dividend and divisor by the same number does not alter the quotient.

WRITTEN EXERCISES.

136. Divide 1326 by 10; by 100.

$$1326 \div 10 = 132.6$$
$$1326 \div 100 = 13.26$$

Solution.—1326 is the same as 132 tens and 6 remainder, and also the same as 13 hundreds and 26 remainder. Hence,

1 ten is in 132 tens, 132 times; the division of the 6 may be denoted by $\frac{6}{10}$, or .6; and the quotient is 132.6.

1 hundred is in 13 hundred 13 times; the division of the 26 may be denoted by $\frac{26}{100}$, or .26; and the quotient is 13.26. That is,—

77. When the divisor is 10, 100, etc.

The quotient may be obtained by simply moving the decimal point in the dividend as many places to the left as there are ciphers in the divisor.

Divide —

137. 2975 by 10.
138. 4576 by 100.
139. 1901 by 100.
140. 1865 by 1000.

141. At $ 10 a barrel, how many barrels of flour can be purchased for $ 1440 ?

142. How many watches, at $ 100 each, can be bought for $ 97800 ?

143. If 100 men share equally a prize of $ 17650, what is each man's share ?

144. Divide 5832 by 60.

$$6|0)\ \underline{583|2}$$
$$97\tfrac{12}{60}$$

Solution. — 60 is the same as 6 tens, and 5832 is the same as 583 tens and a remainder 2 ones. We, therefore, may shorten the process of getting the quotient. Since dividing both the divisor and dividend by the same number will not alter the quotient (Art. 76), we divide both by 10, by marking off in each an order at the right, and the divisor becomes 6 tens and the dividend 583 tens, with the remainder 2.

Dividing now, we have the quotient 97 and a remainder 1 ten. Uniting the first remainder, 2 ones, with the last remainder, 1 ten, we have for the exact remainder 12. Hence, the quotient is $97\tfrac{12}{60}$.

145. If a person can copy 50 words in a minute, how many minutes will he be copying 47569 words ?

146. When 110 plows are worth $ 1320, what is each plow worth ?

147. The distance from the earth to the moon is 240000 miles; how many hours would a person be in reaching it, supposing he could move regularly 30 miles an hour?

148. In how many years will a clerk at a salary of $700 a year earn $9100?

149. 29435 ÷ 176 = ?

150. 34968 ÷ 294 = ?

151. 30769 ÷ 342 = ?

152. 84795 ÷ 694 = ?

153. 87105 ÷ 836 = ?

154. 894325 ÷ 982 = ?

155. 468138 ÷ 593 = ?

156. 564291 ÷ 89 = ?

157. 123456 ÷ 397 = ?

158. 789012 ÷ 846 = ?

159. 345678 ÷ 982 = ?

160. 901234 ÷ 347 = ?

161. 567895 ÷ 563 = ?

162. 456780 ÷ 940 = ?

163. 932057 ÷ 893 = ?

164. 849632 ÷ 842 = ?

QUESTIONS.

65. What is division? 66. What is the dividend? 67. The divisor? 68. The quotient? 69. The remainder?

70. What is the sign of division? 71. A principle of division?

72. How may equal parts of a number be expressed in figures? What may the number written above the line be considered? The number below the line? 73. What is an integer?

74. How are the divisor and dividend written for dividing? How do you begin to divide? For what is the first result written? What is the next step? How long do you divide and proceed as before? If there is a final remainder, what is to be done?

75. What is the proof of division? 76. What is the principle in relation to dividing both dividend and divisor by the same number? 77. When the divisor is 10, 100, etc., how may the quotient be obtained?

UNITED STATES MONEY.

78. United States Money is the currency of the United States. Of this money,

10 mills are 1 cent, marked c., or ¢.
10 cents " 1 dime, " d.
10 dimes " 1 dollar, " $.

79. Coins are pieces of metal stamped by the government to circulate as money.

The principal coins of the United States are:

BRONZE — cent. NICKEL — five-cent. SILVER — dime, quarter-dollar, half-dollar, dollar. GOLD — dollar, quarter-eagle, three-dollar, half-eagle, eagle, and double-eagle. An eagle is ten dollars.

80. Eagles and dollars are expressed together as dollars, and dimes and cents as cents.

Thus, 2 eagles 3 dollars 7 dimes 5 cents are written, $23.75, and read twenty-three dollars and seventy-five cents.

Mills are often expressed as parts of a cent.

Thus, instead of $0.625, may be written $0.62½.

WRITTEN EXERCISES.

1. Change 25 cents to mills.

$25 \times 10 = 250$

Solution. — 1 cent is 10 mills, and 25 cents must be 25 times 10 mills, or 250 mills.

2. Change 67 cents to mills.

3. Change $47 to cents; to mills.

$47 \times 100 = 4700$

$47 \times 1000 = 47000$

Solution. — $1 is 100 cents and $47 must be 47 times 100 cents, or 4700 cents. $1 is 100 times 10 mills, or 1000 mills, and $47 must be 47 times 1000 mills, or 47000 mills.

4. Change $190 to cents.

5. Change $24 to mills.

6. How many cents in 250 mills.

$250 \div 10 = 25$

Solution. — 10 mills are 1 cent, and in 250 mills there must be as many cents as there are times 10 mills in 250 mills, or 25.

7. How many dollars in 900 cents. In 9000 mills.

$$900 \div 100 = 9$$
$$9000 \div 1000 = 9$$

Solution. — 100 cents are $1; and in 900 cents there must be as many dollars as there are times 100 cents in 900 cents, or 9.

1000 mills are $1; and in 9000 mills there must be as many dollars as there are times 1000 mills in 9000 mills, or 9.

8. Change $16 to cents.

9. Change $31.63 to mills.

81. From the preceding, we have the

Rules.

To change cents to mills, move the decimal point one place to the right; dollars to cents, move the decimal point two places to the right; and dollars to mills, move the decimal point three places to the right.

To change mills to cents, move the decimal point one place to the left; cents to dollars, move the decimal point two places to the left; and mills to dollars, move the decimal point three places to the left.

Change —

10. 97 cents to mills.

11. $63 to cents.

12. $9.73 to cents.

13. $18.05 to mills.

14. 970 mills to cents.

15. 6300 cents to dollars.

16. 973 cents to dollars.

17. 18050 mills to dollars.

18. In $1.62 how many mills?

19. In $93.78 how many cents?

20. In 6440 cents how many dollars?

21. In 6780 mills, how many dollars?

22. In 1400 cents, how many dollars.

ORAL EXERCISES.

23. In $ 3 how many dimes.

24. I have $ 1.50 and wish to divide it among 3 boys; how many dimes can I give each?

25. In a box containing 20 cents, John put 14 cents, Mary 5, and Lucy 8. How many cents were there in the box?

26. John had 21 cents; he lost 4 cents and his mother gave him 5; how many had he then?

27. A man paid $ 72 for a watch, and one-eighth as much for a chain; what was the cost of both?

28. If apples cost two cents apiece and peaches 4 cents apiece, what is the cost of 4 apples and 10 peaches?

29. A boy who had 50 cents paid 10 cents for marbles, 8 cents for a top, lost 3 cents, and bought 8 oranges at 3 cents apiece. How many cents did he have left?

30. How many dollars in 450 cents? In 500 cents?

31. How are dollars and cents in the same expression separated or distinguished?

32. A man having $ 47 buys 6 cords of wood at $ 7 per cord; how much money has he left?

33. Mills are reduced to dollars by pointing off three orders of figures. Why?

34. Bought 12 yards of cloth for $ 1.44. What was the cost per yard?

WRITTEN EXERCISES.

35. Write and add: Five dollars and four cents, eighteen dollars and eighty cents, and fifty-three dollars.

36. Add $ 960.17, $ 817.05, $ 3.09, and $ 29.19?

37. Bought a coat for $ 12.50, a vest for $ 6, a pair of shoes for $ 2.12½, and a handkerchief for $ 0.87½; how much did the whole cost?

38. If I give for a plow $ 10.63, for a wagon $ 86.75, for a load of hay $ 17.15, and for a hoe $ 1.25, how much do they all cost me?

39. Johnson owes A $ 145.91, B $ 1690.84, C $ 345.11, D $ 918.85, and E $ 1144.75; what is the whole amount of his debts?

40. Sold a horse for $ 208, a cow for $ 70, and some hay for $ 27.50; how much did the whole bring?

41. A merchant owes three hundred seven dollars, six hundred twenty-two dollars eighty-eight cents, sixty-two dollars 17 cents 5 mills, 325 dollars 12 cents 5 mills: how much does he owe?

42. Subtract $ 59.23 from $ 319.07.

43. Subtract $ 87.62½ from $ 93.08.

44. I had in my pocket $ 102.50, but I spent in a store $ 67.85; how much had I then left?

45. From $ 7 take 7 cents.

46. From $ 11 take 1 mill.

47. Bought goods for $ 5675.50, and sold them for $ 7500.31; how much was made by the operation?

48. What is the product of $ 192.075 by 108.

49. Multiply $ 47.56 by 16.

50. What cost 9 tons of coal at $ 5.50 per ton?

51. What will be the cost of 563 bushels of wheat at $ 2.25 per bushel?

52. If $ 11.44 be paid for 26 yards of drilling, what is the cost per yard?

53. Divide $ 355 equally among 25 men, carrying the division to cents.

```
     $ 14.20
25) $ 355.00
      25
      ---
      105
      100
      ---
        50
        50
        --
         0
```

Solution. — Since the division is to be carried to cents, we annex to the dividend two decimal orders, for the cents' places, by writing at the right of the decimal point two zeros; then dividing we have as the result required $ 14.20.

54. Paid $ 78 for 12 sheep; how much was that for each?

55. J. Smith paid $ 255 for 75 sheep. How much was that a head?

56. Divide $ 497 by 4, carrying the division to cents?

57. Paid $ 240 for 32 tons of coal. How much was that a ton?

58. Divide $ 2764 by 24 to mills? Ans. $ 115.166+.

Here, in the answer, the division not being exact, the sign + is used to show that the division might be carried further.

59. Divide $ 637 among 27 men, to cents.

60. Paid $ 5.27 for 17 boxes of choice strawberries; what was that a box?

61. How much must be paid for 115 tons of coal at $ 8.75 a ton?

62. A man bought a horse for $ 150, and 2 yoke of oxen at $ 175.50 each. How much did the whole cost him?

63. At 62 cents a bushel, how many bushels of corn can be bought for $ 31.

Solution. — $ 31 = 3100 cents; 3100 ÷ 62 cents = 50, the number of bushels required.

64. Paid $ 17.10 for coffee at 38 cents a pound. How many pounds did I buy?

65. How many bushels of wheat at $ 1.05 a bushel can be bought for $ 71.40?

QUESTIONS.

78. What is United States money? Of this money how many mills are 1 cent; how many cents 1 dime; etc.? How are eagles and dollars expressed? How are dimes and cents expressed?

79. What are coins? What coin is of bronze? What coin is of nickel? What of silver? What of gold?

81. How are cents changed to mills? Dollars to cents? Dollars to mills? Mills to cents? Cents to dollars? Mills to dollars?

REVIEW.

ORAL EXERCISES.

82. 1. How many times 7 + 2 in 72? In 81?

2. How many are 6 times 8 − 3? 8 times 9 − 5?

3. To 6 add 4, subtract 3, multiply by 9, divide by 7. What is the result?

4. From 19 subtract 11, multiply by 10, add 16, divide by 12. What is the result?

5. Multiply 12 by 6, divide by 8, add 5, multiply by 3, subtract 12. What remains?

6. The multiplier is 8, and the product 96. What is the multiplicand?

7. The sum of three numbers is 51, and two of the numbers are 16 and 11. What is the third number.

8. The minuend is 93 and the difference 79. What is the subtrahend?

9. Divide 81 by 9, add 6, multiply by 4, and add 17. What is the result?

10. The integer of the quotient is 7, the remainder 9, and the divisor, 12. What is the dividend?

11. A boy had 49 cents and his father gave him 11. He then spent his money for 4 doves. How much did he give for each?

12. A farmer raised 84 bushels of corn; saving 12 bushels for his own use, he sold 7 bushels to his brother; and sold the rest in equal quantities to 5 persons. How much did he sell each of the 5 persons?

13. If you should sell 10 tons of coal at $ 6 a ton, how much cloth at $ 4 a yard could you buy with the money?

14. If 7 barrels of flour cost $ 56, what will 1 barrel cost? What will 4 barrels cost?

15. If 4 tons of hay cost $ 48, what will 7 tons cost?

16. If 9 quarts of milk cost 54 cents, what will 7 quarts cost?

17. If 15 yards of cloth cost $ 75, what will 60 yards cost?

18. When 4 quarts of vinegar can be bought for 28 cents, how many quarts can be bought for 84 cents?

19. If a steamer goes 75 miles in 5 hours, in how many hours can she go 90 miles?

20. Received for 4 bushels of potatoes 28 pounds of pork; how many pounds should I receive for 9 bushels?

21. A and B start together and travel in the same direction along the same road. A goes 20 miles a day and B 28 miles. When B shall be 80 miles ahead of A, how many days will they have travelled?

83. A **Parenthesis,** ( ), or **Vinculum,** ——, is used to include such numbers as are to be considered together.

Thus, $(8 + 5 - 3) \times 6$, or $\overline{8 + 5 - 3} \times 6$, denotes that 8 and 5, less 3, or 10, is to be multiplied by 6.

WRITTEN EXERCISES.

22. Express in words the number denoted by 237321.

23. Express in figures one hundred seventy-nine dollars and seven cents five mills.

24. Add together ninety thousand three hundred forty, one thousand three hundred six, and two.

25. The smaller of two numbers is 569, and their difference is 731. What is the larger number?

26. Jones is worth four thousand twenty-five dollars, and Ford three thousand nine hundred dollars and eighty-seven cents. How much is Jones worth more than Ford?

27. Show by an example how subtraction is proved by addition.

28. A product is 4680, and one of its two factors is 15. What is the other?

29. Bought goods to the amount of $ 34.67, paying for them in corn $ 4.21, in wheat $ 9.68, and the rest in money. How much money was paid?

30. What is the value of $(3604 + 162) \times 3$?

31. From $(612 + 12)$ take $(316 - 32)$.

32. Smith's farm is worth thirteen thousand four hundred sixty dollars, his cattle six hundred forty-three dollars fifty cents, and his farming tools two hundred three dollars and eighty-eight cents. How much is the whole worth?

33. Gave $ 3375 and 5 horses worth each $ 125, for a farm of 160 acres. What was the cost of the farm per acre?

34. Show by an example how multiplication and division prove each other?

35. Bought a bill of goods, and on paying gave a five hundred dollar note. The change paid back was $ 169.73; what was the amount of the bill?

36. (565 + 10) ÷ 25 = how many?

37. Bought 3 barrels of flour at $ 12 a barrel, and 6 tons of hay at $ 30 a ton, and paid for the whole 54 sheep. What was the value of each sheep?

38. 8 + 5, − 3, × 9, + 6, ÷ 12; find the result.

39. 24 − 8, + 7, × 10, − 7, ÷ 8; find the result.

40. (60 × 5) + 50 ÷ 25 = ?

41. Find the quotient of 84693 ÷ 89.

42. At $ 85 an acre, how many acres of land can be bought for $ 8675, and what will be left?

43. How many yards of broadcloth at $ 6.50 a yard can be bought for $ 201.50.

44. Bought a farm of 110 acres for $ 6875. How much was it per acre?

45. A father died leaving $ 12560, which was divided equally among 8 heirs. How much was each heir's share?

46. Bought a box of raisins for $ 3.15, a tub of butter for $ 14.50, and flour for $ 53.65. The whole was sold at a profit of $ 9.65. What sum did the whole bring?

47. How many yards of cloth at $ 3 a yard will pay for 20 tons of coal at $ 7.50 a ton?

48. How many pounds of butter at 37½ cents a pound can be bought for $ 15?

49. Bought a farm containing 40 acres of meadow and 17 of woodland, for $ 2850. Sold to one man 10 acres of woodland for $ 85 per acre; to another a house lot of one acre for $ 90 and the remainder to a third for $ 2025. What did I gain per acre by the operation?

50. A drover has $ 2144, with which he wishes to buy horses at $ 165 each, and then lay out the remainder for sheep at $ 4 each. How many of each can he buy?

REVIEW QUESTIONS.

1. What is a unit? 2. What is a number? 4. What are figures? 5. How many are used in expressing numbers?

7. In naming numbers what are nine and one called? 9. What are ten tens called? 11. Ten hundreds?

21. What is a group? Of what is the group of ones composed? The group of thousands? 26. Name the groups of the numeration table from ones to billions? Name the orders of units of the table from ones to hundreds of millions.

22. What are decimals? 23. How are they distinguished? Name the three successive orders at the right of the decimal point.

39. How many and what decimal places do cents, when written in figures, occupy? What decimal place do mills, when written in figures, occupy?

71. What terms in division correspond to the multiplier and multiplicand in multiplication? What term in division to the product in multiplication? How then would you infer that multiplication can be proved?

83. What is the parenthesis, or vinculum, used to include?

COMMON FRACTIONS.

84. **1.** If an apple is cut into two equal parts, what part of the apple will be one of the equal parts?

2. Edward has separated an orange into three equal parts. What part of the orange is one of the equal parts? What part of the orange is two of the equal parts?

3. Alice has cut a cake into four equal parts. What part of the cake is one of the equal parts? Is two of the equal parts? Is three of the equal parts?

4. How many halves in an apple? How many thirds in an orange?

5. How many halves in any thing? How many thirds in any thing?

6. How many fourths in a cake? How many fourths in any thing?

7. Into how many halves can a single thing or unit be divided? Into how many thirds? Into how many fourths?

85. A **Fraction** is one or more of the equal parts of a unit.

86. The **Denominator** of a fraction is the number which shows into how many equal parts the unit is divided.

Thus, three is the denominator of two thirds.

87. The **Numerator** of a fraction is the number which shows how many of the equal parts of the unit are taken.

Thus, two is the numerator of two thirds.

88. A **Common Fraction** is a fraction expressed by writing the numerator above, and the denominator below, a dividing line; as,

One half,	written	$\frac{1}{2}$	Two fifths,	written	$\frac{2}{5}$
One third,	"	$\frac{1}{3}$	Three fifths,	"	$\frac{3}{5}$
Two thirds,	"	$\frac{2}{3}$	Four fifths,	"	$\frac{4}{5}$
One fourth,	"	$\frac{1}{4}$	One seventh,	"	$\frac{1}{7}$
Two fourths,	"	$\frac{2}{4}$	Three eighths,	"	$\frac{3}{8}$
Three fourths,	"	$\frac{3}{4}$	Five twenty-firsts,	"	$\frac{5}{21}$
One fifth,	"	$\frac{1}{5}$	Seven forty-seconds,	"	$\frac{7}{42}$

89. An **Integer** (Art. 73) may be expressed in a fractional form, by writing 1 under it, for a denominator. Thus,

2 may be written $\frac{2}{1}$, and read, 2 ones.
3 may be written $\frac{3}{1}$, and read, 3 ones.
7 may be written $\frac{7}{1}$, and read, 7 ones.
10 may be written $\frac{10}{1}$, and read, 10 ones.

90. A **Mixed Number** is an integer and a fraction.

Thus, $3\frac{1}{4}$, read three and one fourth, is a mixed number.

91. A **Fraction** may be regarded as an expression of division (Art. 72), the numerator answering to the dividend, and the denominator to the divisor. Hence,

92. The **Value of a Fraction is** the quotient of the numerator divided by the denominator.

Thus, $\frac{3}{4}$ of 1 inch is an expression of $\frac{1}{4}$ of 3 inches.

WRITTEN EXERCISES.

Write and read —

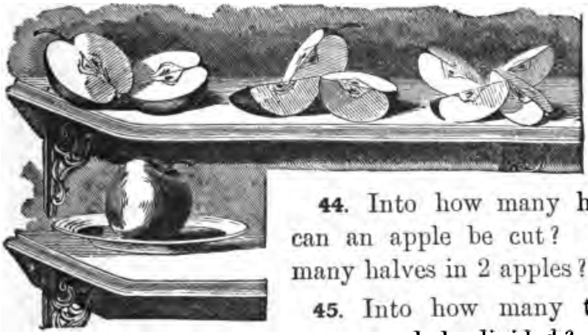
8. $\frac{3}{4}$.
9. $\frac{5}{9}$.
10. $\frac{7}{11}$.
11. $\frac{8}{15}$.
12. $\frac{10}{1}$.
13. $\frac{16}{40}$.
14. $\frac{19}{21}$.
15. $\frac{13}{15}$.
16. $\frac{17}{10}$.
17. $\frac{11}{13}$.
18. $\frac{9}{4}$.
19. $\frac{3}{2}$.
20. $\frac{34}{16}$.
21. $\frac{17}{1}$.
22. $\frac{14}{5}$.
23. $\frac{81}{1}$.
24. $\frac{60}{71}$.
25. $\frac{5}{11}$.
26. $\frac{7}{72}$.
27. $\frac{11}{41}$.

Express in figures:

28. Two sixths.
29. Three sevenths.
30. Five eighths.
31. Six ninths.
32. Two tenths.
33. Seven elevenths.
34. Eight twelfths.
35. Nine sixteenths.
36. Seventeen ones.
37. One twenty-first.
38. Sixteen sevenths.
39. Nine forty-seconds.
40. Five and two-thirds.
41. Seventeen fortieths.
42. Nineteen twelfths.
43. Sixty and three-eighths.

REDUCTION OF FRACTIONS.

Integers and Mixed Numbers to Fractions.



44. Into how many halves can an apple be cut? How many halves in 2 apples?

45. Into how many thirds can an apple be divided? How many thirds in 2 apples? In 3 apples?

46. Into how many fourths can an apple be divided? How many fourths in 3 apples? In 5 apples?

47. How many halves in 2? Thirds in 2? Fourths in 3?

48. How many fifths in 1 melon? In 2 melons? In 5 melons?

49. How many eighths in 1? In 4? In 7?

50. Jane has $2\frac{2}{3}$ apples. How many thirds of an apple has she?

Solution. — In 1 apple there are 3 thirds, and in 2 apples there must be 2 times 2 thirds, or 6 thirds; 6 thirds and 2 thirds are 8 thirds. Ans. $\frac{8}{3}$.

51. How many fourths of an apple in $1\frac{3}{4}$ apples. In $2\frac{1}{4}$ apples?

52. In $2\frac{1}{3}$ how many thirds? In $6\frac{1}{4}$ how many fourths? In $7\frac{2}{9}$ how many ninths?

53. In 6 how many halves? In $11\frac{1}{3}$ how many thirds? In $7\frac{3}{4}$ how many fourths?

54. Change 7 to fourths? $9\frac{1}{2}$ to halves?

55. Change 8 to a fraction whose denominator is 3. 13 to a fraction whose denominator is 1.

93. A **Proper Fraction** is one whose numerator is less than its denominator.

Thus, $\frac{5}{8}$ and $\frac{7}{11}$ are proper fractions.

94. An **Improper Fraction** is one whose numerator is not less than its denominator.

Thus, $\frac{5}{5}$ and $\frac{13}{11}$ are improper fractions.

95. **Reduction of Fractions** is changing their form without changing their value.

WRITTEN EXERCISES.

56. Change 18 to sevenths, or to a fraction whose denominator is 7.

$$\begin{array}{ll} 1 = \frac{7}{7}. & 18 \\ & \underline{7} \\ & 126 \\ 18 = & \overline{7} \end{array}$$

Solution. — In 1 there are 7 sevenths, and in 18 there are 18 times 7 sevenths, or 126 sevenths. Ans. $\frac{126}{7}$.

57. Change 9 to a fraction whose denominator is 15.

58. Reduce 131 to elevenths.

59. Change $15\frac{2}{3}$ to thirds.

$$\begin{array}{lll} 1 = \frac{3}{3}. & 15 & \\ & \underline{3} & \\ 15 \ = & 45 & \text{thirds.} \\ & \underline{2} & \text{thirds.} \\ 15\frac{2}{3} = & \frac{47}{3}, & \text{Ans.} \end{array}$$

Solution. — In 15 there are 15 times 3 thirds, or 45 thirds; which with 2 thirds added are 47 thirds. Ans. $\frac{47}{3}$.

60. In $\$10\frac{1}{4}$ how many fourths of a dollar?

61. Change $19\frac{1}{3}$ to an improper fraction.

96. To reduce integers or mixed numbers to fractions.

Rule.

Multiply the integer by the denominator, and if there is a fractional part, add its numerator to the product; and write the result over the denominator.

Change, —

62. 14 to fourths.

63. 19 to fifths.

64. 31 to eighths.

65. 98 to thirds.

66. 70 to thirty-seconds.

67. 135 to seventieths.

Reduce to improper fractions,

68. $7\frac{4}{9}$.	**70.** $9\frac{2}{11}$.	**72.** $12\frac{15}{17}$.	**74.** $19\frac{13}{18}$.
69. $6\frac{4}{13}$.	**71.** $16\frac{3}{7}$.	**73.** $14\frac{3}{10}$.	**75.** $161\frac{21}{40}$.

Fractions to Integers or Mixed Numbers.

76. How many apples in 2 halves of an apple? In 3 thirds? In 4 fourths?

77. How many dollars in 11 fourths of a dollar?

Solution. — In 11 fourths of a dollar there are as many dollars as 4 fourths of a dollar are contained times in 11 fourths of a dollar, or $2\frac{3}{4}$. Ans. \$$2\frac{3}{4}$.

78. How many feet in 13 thirds of a foot? In 17 thirds of a foot?

79. How many dollars in \$$\frac{19}{8}$? In \$$\frac{81}{10}$? In \$$\frac{43}{12}$?

80. Change to integers $\frac{16}{4}$; $\frac{21}{7}$; $\frac{64}{8}$; $\frac{81}{9}$; $\frac{110}{10}$; $\frac{108}{12}$.

81. Change to mixed numbers $\frac{37}{7}$; $\frac{53}{8}$; $\frac{67}{9}$; $\frac{93}{10}$; $\frac{79}{11}$; $\frac{109}{12}$; $\frac{103}{25}$.

WRITTEN EXERCISES.

82. Change $\frac{793}{19}$ to a mixed number.

$$\begin{array}{r} 41\frac{14}{19} \\ 19\overline{)\,793} \\ \underline{76} \\ 33 \\ \underline{19} \\ 14 \end{array}$$

Solution. — As 19 nineteenths are 1, 793 nineteenths will be as many ones, as times 19 in 793, or $41\frac{14}{19}$. Ans. $41\frac{14}{19}$.

83. Change $\frac{623}{7}$ to an integer or mixed number.

84. Change $\frac{572}{13}$ to an integer or mixed number.

85. Change $\frac{931}{17}$ to an integer or mixed number.

97. To change an improper fraction to an integer or mixed number.

Rule.

Divide the numerator by the denominator.

Reduce to an integer or mixed number:

86. $\frac{67}{9}$.
87. $\frac{98}{7}$.
88. $\frac{108}{4}$.
89. $\frac{313}{13}$.
90. $\frac{405}{9}$.
91. $\frac{405}{9}$.
92. $\frac{614}{19}$.
93. $\frac{106}{106}$.
94. $\frac{185}{23}$.

95. What is the value of $\frac{473}{55}$ tons?

96. What is the value of $\frac{8224}{16}$ miles?

A Fraction to its Smallest Terms.

97. In 4 fourths of 1 apple, how many halves of an apple? If you divide both the numerator and the denominator of $\frac{4}{4}$ by 2, what will you have?

98. In 6 sixths of 1 cake, how many halves of a cake? How many thirds of a cake? If you divide both the numerator and the denominator of $\frac{6}{6}$ by 2, what do you have? By 3, what do you have?

99. In 4 sixths of a cake, how many thirds of a cake?

Solution. — In 2 sixths of a cake there is 1 third of a cake, and in 4 sixths there are as many thirds as times 2 sixths in 4 sixths, or 2. In 4 sixths of a cake there are $\frac{2}{3}$ of a cake.

100. Change $\frac{4}{6}$ to thirds, $\frac{6}{8}$ to fourths, $\frac{6}{12}$ to sixths.

101. How many fourths in 2 eighths? In $\frac{4}{8}$? In $\frac{6}{8}$?

102. How many fifths in $\frac{6}{30}$? In $\frac{12}{30}$? In $\frac{24}{30}$?

103. Change $\frac{6}{9}$ to thirds; $\frac{12}{15}$ to fifths; $\frac{15}{21}$ to sevenths.

104. If you divide both the numerator and the denominator of $\frac{12}{15}$ by 3, what do you have?

105. By what integer greater than 1 can you divide both the numerator and denominator of $\frac{15}{20}$ without a remainder?

98. A **Common Divisor** of two or more numbers is any integer above one that will exactly divide each of them.

NOTE.—A common divisor of two or more numbers is also a factor (Art. 56), common to those numbers.

99. The **Terms** of a fraction are its numerator and denominator.

100. A Fraction is in its **Smallest Terms** when its terms have no common divisor.

101. A Principle of Fractions.

Multiplying or dividing both terms of a fraction by the same number does not change its value.

WRITTEN EXERCISES.

106. Reduce $\frac{18}{24}$ to its smallest terms.

$$\frac{2)\,18}{2)\,24} = \frac{9}{12}$$

$$\frac{3)\,9}{3)\,12} = \frac{3}{4}$$

Solution.—Dividing both terms of $\frac{18}{24}$ by 2, we have $\frac{9}{12}$; and dividing both terms of $\frac{9}{12}$ by 3, we have $\frac{3}{4}$, whose terms have no common divisor. $\frac{18}{24}$ changed to its smallest terms is $\frac{3}{4}$.

107. Reduce to its smallest terms $\frac{81}{108}$.

108. Reduce to its smallest terms $\frac{120}{144}$.

102. To reduce a fraction to its smallest terms.

Rule.

Divide the terms of the fraction by any common divisor, and continue to divide thus until they have no common divisor.

NOTE. — The greater the common divisor used, the shorter will be the process.

Reduce to their smallest terms:

109. $\frac{90}{120}$.
110. $\frac{77}{88}$.
111. $\frac{132}{264}$.
112. $\frac{49}{343}$.
113. $\frac{147}{168}$.
114. $\frac{182}{196}$.
115. $\frac{252}{308}$.
116. $\frac{60}{125}$.
117. $\frac{72}{94}$.

118. Find the smallest terms of $\frac{840}{1120}$.

Fractions to a Common Denominator.

119. How many fourths of an inch in 1 half? In 2 halves?

One Inch.

120. If an apple is divided into 3 equal parts, and each of the parts into 2 equal pieces, what part of the apple will one of these equal pieces be?

121. In half of any thing how many fourths? How many sixths?

122. How many sixths of a cake in $\frac{1}{3}$ of a cake? In $\frac{2}{3}$ of a cake?

Solution. — In 1 third of a cake there are 2 sixths, and in 2 thirds there are 2 times 2 sixths, or 4 sixths. Ans. $\frac{4}{6}$.

123. How many eighths in $\frac{1}{4}$? In $\frac{2}{4}$? In $\frac{3}{4}$?

124. How many tenths in $\frac{1}{5}$? In $\frac{2}{5}$? In $\frac{3}{5}$?

125. How many twelfths in $\frac{1}{6}$? In $\frac{3}{6}$? In $\frac{5}{6}$?

126. How many fourteenths in $\frac{3}{7}$? Sixteenths in $\frac{5}{8}$?

127. Express $\frac{1}{2}$ and $\frac{2}{3}$, each as sixths.

128. Express $\frac{3}{4}$ and $\frac{1}{2}$, each as eighths.

129. Express $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$, each as twelfths.

130. Change $\frac{1}{7}$ and $\frac{1}{5}$ to fractions having the same denominator.

131. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{1}{4}$ to fractions having the same denominator.

103. Fractions have a **Common Denominator** when they have the same number for a denominator.

WRITTEN EXERCISES.

132. Change $\frac{2}{5}$ and $\frac{3}{7}$ to fractions having a common denominator.

$$\frac{2}{5} = \frac{14}{35}$$
$$\frac{3}{7} = \frac{15}{35}$$

Solution. — We multiply both terms of $\frac{2}{5}$ by 7, the denominator of the second fraction (Art. 101), and have $\frac{14}{35}$; and both terms of $\frac{3}{7}$ by 5, the denominator of the first fraction, and have $\frac{15}{35}$. The fractions required are $\frac{14}{35}$ and $\frac{15}{35}$.

133. Change $\frac{5}{6}$ and $\frac{8}{9}$ to fractions having a common denominator.

134. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{12}$ to fractions having a common denominator.

$\frac{2}{3} = \frac{8}{12}$
$\frac{5}{6} = \frac{10}{12}$
$\frac{3}{12}$

Solution. — By multiplying both terms of $\frac{2}{3}$ by 4, and $\frac{5}{6}$ by 2, we have $\frac{8}{12}$ and $\frac{10}{12}$, which have the same denominator as $\frac{3}{12}$. The fractions required are $\frac{8}{12}$, $\frac{10}{12}$ and $\frac{3}{12}$.

135. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{3}{16}$ to fractions having a common denominator.

104. To reduce fractions to a common denominator.

Rule.

Multiply both terms of each fraction by the denominators of the other fractions. Or,

Multiply both terms of one or more of the fractions by such a number as will make all the denominators alike.

Reduce to a common denominator:

136. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

137. $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{1}{24}$.

138. $\frac{2}{5}$, 4, and $\frac{3}{10}$.

139. $\frac{3}{4}$, $\frac{5}{9}$, and $\frac{7}{12}$.

140. $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{4}{15}$.

141. $\frac{3}{8}$, $\frac{2}{16}$, and $\frac{1}{4}$.

142. $2\frac{1}{2}$, 3, $\frac{5}{4}$, and $\frac{3}{8}$.

143. $\frac{5}{6}$ and $\frac{3}{8}$.

144. $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{3}{2}$.

145. James had given him $\frac{3}{5}$ of a melon, John $\frac{1}{5}$ of a melon, and Susan $\frac{1}{6}$. How many 30ths of a melon were given to each of them ?

ADDITION OF FRACTIONS.

146. If Henry has $\frac{3}{5}$ of a dollar and Arthur $\frac{4}{5}$, how many fifths have both?

147. How many fourths are $\frac{1}{4}$ and $\frac{1}{2}$?

148. How many eighths are $\frac{3}{4}$ and $\frac{5}{8}$?

149. If you should give your brother $\frac{3}{8}$ of a melon, your sister $\frac{1}{8}$, and a playmate $\frac{2}{8}$, what part of the melon would you give away?

150. How many eighths are $\frac{1}{8} + \frac{2}{8} + \frac{3}{8}$?

151. How many are $\frac{5}{6} + \frac{2}{6}$? $\frac{4}{7} + \frac{3}{14}$?

152. How many are $\frac{1}{2} + \frac{1}{3}$? $\frac{2}{3} + \frac{1}{6}$?

153. A farmer has in one garden $\frac{10}{16}$ of an acre, and in another $\frac{5}{8}$ of an acre. What is the extent of the two gardens?

WRITTEN EXERCISES.

154. What is the sum of $\frac{1}{5} + \frac{3}{5} + \frac{4}{5}$?

$$\frac{1}{5} + \frac{3}{5} + \frac{4}{5} = \frac{8}{5} = 1\frac{3}{5}$$

Solution.—The sum of $\frac{1}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$ is $\frac{8}{5}$, or $1\frac{3}{5}$.

155. What is the sum of $\frac{11}{12} + \frac{9}{12} + \frac{7}{12}$?

156. What is the sum of $\frac{3}{4}$, $\frac{4}{5}$, and 2?

$$\frac{3}{4} + \frac{4}{5} + \frac{2}{1} =$$
$$\frac{15}{20} + \frac{16}{20} + \frac{40}{20} =$$
$$\frac{71}{20} = 3\frac{11}{20}$$

Solution.— Changing the fractions to twentieths, and adding the numerators, we have $\frac{71}{20} = 3\frac{11}{20}$ as the required sum.

157. What is the sum of $\frac{5}{6}$, $\frac{3}{5}$, and $\frac{1}{2}$?

105. To add fractions:

Rule.

Change the fractions, if necessary, to fractions having a common denominator; write the sum of the new numerators over the common denominator, and simplify the result.

Find the sum of —

158. $\frac{4}{9}$, $\frac{7}{9}$, and $\frac{11}{9}$.

159. $\frac{8}{15}$ and $\frac{8}{20}$.

160. $\frac{3}{5}$ and $\frac{4}{7}$.

161. $\frac{4}{7}$, $\frac{2}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$.

162. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$, and $\frac{1}{4}$.

163. $\frac{1}{8}$, $\frac{3}{10}$, and $\frac{4}{12}$.

164. $\frac{7}{10}$, $\frac{11}{12}$, and $\frac{4}{9}$.

165. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{7}$.

166. $\frac{1}{8}$, $\frac{2}{3}$, and $\frac{3}{4}$.

167. $\frac{4}{5}$, $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{7}{10}$.

106. When there are mixed numbers, *the fractions and the integers may be added separately, and then the results added.*

168. What is the sum of $5\frac{2}{3}$, $6\frac{7}{8}$, and $4\frac{1}{2}$?

169. What is the sum of $17\frac{1}{2}$, $2\frac{2}{2}$, and 3?

170. Required the sum of $5\frac{4}{9}$, 6, and $7\frac{3}{7}$.

171. What is the sum of \$ $9\frac{1}{5}$, \$ $12\frac{1}{3}$, \$ $10\frac{2}{3}$, and \$ $7\frac{3}{4}$?

172. I have bought three lots of coal, weighing respectively, $1\frac{3}{5}$, $\frac{4}{15}$, and $9\frac{3}{20}$ tons. How much is there in all?

173. Bought a handkerchief for \$ $\frac{5}{8}$, a vest for \$ $2\frac{1}{2}$, a pair of gloves for \$ $\frac{5}{16}$, and a hat for \$ $4\frac{1}{4}$. How much did the whole cost?

174. A merchant has sold from a piece of cloth $4\frac{2}{3}$, $8\frac{1}{7}$, and $2\frac{5}{7}$ yards. How many yards is this in all?

SUBTRACTION OF FRACTIONS.

175. David had \$ $\frac{4}{5}$ and gave away \$ $\frac{3}{5}$. How many fifths of a dollar had he left?

176. How much is $\frac{4}{5}$ less $\frac{3}{5}$? $\frac{4}{5}$ less $\frac{2}{5}$?

177. If you should have $\frac{5}{6}$ of a cake and should give your brother $\frac{2}{6}$, how many sixths of a cake would you have left?

178. How much is $\frac{5}{6} - \frac{2}{6}$? $\frac{6}{7}$ less $\frac{3}{7}$?

179. A man owned $\frac{7}{8}$ of a boat and sold $\frac{1}{2}$ of the boat. How many eighths of the boat had he left.

Solution. — $\frac{1}{2} = \frac{4}{8}$, and $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$, Ans.

180. How much is $\frac{7}{8}$ less $\frac{1}{2}$? 1 less $\frac{2}{3}$? $\frac{7}{9}$ less $\frac{5}{18}$?

181. How much is $\frac{11}{12}$ less $\frac{1}{3}$? $\frac{11}{12}$ less $\frac{3}{4}$? $\frac{15}{16}$ less $\frac{7}{8}$?

182. How much is $3\frac{1}{3}$ or $\frac{10}{3}$ greater than $\frac{5}{6}$? $4\frac{1}{5}$ than $\frac{3}{4}$?

183. Bought 2 tons of coal and sold $1\frac{5}{8}$ tons. How much was left?

WRITTEN EXERCISES.

184. What is the difference between $\frac{5}{13}$ and $\frac{2}{7}$?

$$\frac{5}{13} - \frac{2}{7} =$$
$$\frac{35}{91} - \frac{26}{91} = \frac{9}{91}$$

Solution. — Reducing the given fractions to equivalent fractions having 91 for a denominator, and finding the difference of the numerators, we have as the difference required $\frac{9}{91}$.

185. What is the difference between $\frac{97}{108}$ and $\frac{69}{108}$?

186. What is the difference between $\frac{15}{16}$ and $\frac{4}{9}$?

107. To subtract one fraction from another:

Rule.

Reduce the fractions, if neccssary, to fractions having a common denominator, and then write the difference of the new numerators over the common denominator.

What is the difference between —

187. $\frac{5}{6}$ and $\frac{4}{5}$. **190.** $\frac{7}{8}$ and $\frac{3}{8}$. **193.** $\frac{11}{12}$ and $\frac{3}{4}$.
188. $\frac{8}{15}$ and $\frac{9}{20}$. **191.** $\frac{1}{7}$ and $\frac{1}{11}$. **194.** $\frac{15}{16}$ and $\frac{4}{7}$.
189. $\frac{3}{8}$ and $\frac{1}{2}$. **192.** $\frac{15}{16}$ and $\frac{11}{12}$. **195.** $\frac{16}{21}$ and $\frac{8}{15}$.

196. From $\frac{3}{5}$ subtract $\frac{2}{7}$.

197. How much is $\frac{5}{9}$ greater than $\frac{4}{21}$?

198. Gave $\frac{5}{6}$ of a dollar for a book, and $\frac{2}{5}$ of a dollar for a slate; how much more did the book cost than the slate?

199. What is the difference between 7 and $\frac{3}{4}$?

$$7 - \frac{3}{4} = \frac{28}{4} - \frac{3}{4} = \frac{25}{4} = 6\frac{1}{4}$$

Solution. — $7 = \frac{28}{4}$; and $\frac{28}{4}$ less $\frac{3}{4} = \frac{25}{4}$, or $6\frac{1}{4}$.

200. Subtract $2\frac{6}{7}$ from 5. **201.** Subtract $5\frac{1}{2}$ from 9.

108. When there are mixed numbers, *the fractions and integers may be subtracted separately, and the results united.*

202. What is the difference between $6\frac{3}{4}$ and $5\frac{3}{8}$.

203. Subtract $2\frac{1}{2}$ from $5\frac{5}{8}$.

204. Subtract $20\frac{1}{5}$ from $29\frac{3}{4}$.

205. Subtract $6\frac{1}{2}$ from $12\frac{5}{12}$.

$$\begin{aligned} 12\tfrac{5}{12} &= 11\tfrac{17}{12} \\ 6\tfrac{1}{2} &= \underline{\;6\tfrac{6}{12}} \\ & \quad\; 5\tfrac{11}{12} \end{aligned}$$

Solution. — As $\frac{6}{12}$ is greater than $\frac{5}{12}$, we take 1, or $\frac{12}{12}$, from 12 leaving 11, and add the $\frac{12}{12}$ to the $\frac{5}{12}$, making $\frac{17}{12}$, and then subtracting the $\frac{6}{12}$ from the $\frac{17}{12}$, and the 6 from the 11, have as the result required $5\frac{11}{12}$.

206. If from a cask containing $25\frac{3}{8}$ gallons of wine, $7\frac{2}{3}$ gallons leak out, how much remains in it?

207. Bought a lot of coal for \$ $31\frac{7}{8}$, and sold it for \$ $37\frac{9}{10}$; how much was gained?

208. Paid \$ $113\frac{7}{8}$ for some goods, and sold them at \$ $7\frac{3}{4}$ under cost; for how much were they sold?

209. When a ton of coal, bought for \$ $5\frac{9}{10}$, is sold for \$ $7\frac{1}{4}$, how much is gained?

210. Paid \$ $10\frac{3}{10}$ for a barrel of flour, and sold it for \$ $1\frac{7}{12}$ less than cost. How much was obtained for it?

MULTIPLICATION OF FRACTIONS.

A Fraction by an Integer.

211. If a boy can earn $\frac{3}{10}$ of a dollar in 1 day, how many tenths of a dollar can he earn in 2 days? In 3 days?

212. How many fifths are 2 times $\frac{2}{5}$? 3 times $\frac{2}{5}$?

213. What part of a melon is 4 times $\frac{3}{13}$ of a melon?

214. An owner of a ship sold $\frac{3}{16}$ of it to each of 5 men. What part of the ship did he sell?

215. How much is 6 times $\frac{3}{7}$?

Solution. — 6 times $\frac{3}{7} = \frac{18}{7} = 2\frac{4}{7}$.

216. How much is 4 times $\frac{7}{32}$? 6 times $\frac{7}{24}$? 7 times $\frac{5}{8}$? 8 times $\frac{6}{7}$? 9 times $\frac{4}{5}$?

217. How much is 8 times $\frac{5}{16}$? 9 times $\frac{3}{27}$? 10 times $\frac{5}{11}$? 12 times $\frac{7}{36}$? 11 times $\frac{3}{10}$?

218. How much is 8 times $3\frac{1}{3}$?

Solution. — 8 times $\frac{1}{3} = \frac{8}{3} = 2\frac{2}{3}$; 8 times 3 = 24; and $24 + 2\frac{2}{3} = 26\frac{2}{3}$.

219. How much is 9 times $5\frac{1}{5}$? 7 times $11\frac{3}{7}$? 8 times $10\frac{5}{6}$? 9 times $2\frac{1}{2}$? 8 times $4\frac{1}{5}$?

220. How much is 8 times $\frac{8}{64}$? 10 times $3\frac{3}{4}$? 11 times $2\frac{2}{7}$? 7 times $\frac{5}{28}$? 9 times $3\frac{1}{4}$?

221. At $5\frac{1}{4}$ a cord, what will 7 cords of wood cost?

222. At $7\frac{3}{8}$ a barrel, how much must be paid for 10 barrels of flour?

109. Principle.

A fraction is multiplied either by multiplying its numerator, or by dividing its denominator.

WRITTEN EXERCISES.

223. How much is $\frac{4}{15}$ multiplied by 3.

$$\frac{4}{15} \times 3 = \frac{4 \times 3}{15} = \frac{12}{15} = \frac{4}{5}$$

Solution. — 3 times $\frac{4}{15} = \frac{4 \times 3}{15} = \frac{12}{15}$, or $\frac{4}{5}$.

224. How much is $\frac{11}{13}$ multiplied by 7.

225. Find the product of $\frac{17}{24}$ multiplied by 12.

110. To multiply a fraction by an integer:

Rule.

Either multiply the numerator of the fraction, or divide the denominator of the fraction, by the integer.

Multiply —

226. $\frac{7}{11}$ by 4.
227. $\frac{8}{16}$ by 7.
228. $\frac{3}{4}$ by 16.
229. $\frac{3}{5}$ by 18.
230. $\frac{7}{12}$ by 11.
231. $\frac{15}{16}$ by 25.
232. $\frac{3}{20}$ by 7.
233. $\frac{18}{11}$ by 5.
234. $\frac{19}{21}$ by 21.

235. Find the product of $\frac{9}{13}$ multiplied by 26.

236. At \$$\frac{7}{8}$ a bushel, what cost 12 bushels of corn?

111. When the multiplicand is a mixed number, *the fraction and the integer of the mixed number may be multiplied separately, and the results added.*

237. What is the product of $7\frac{3}{4}$ by 5?

$$\begin{array}{r} 7\frac{3}{4} \\ 5 \\ \hline 4)\underline{15} \\ 3\frac{3}{4} \\ 35 \\ \hline 38\frac{3}{4} \end{array}$$

Solution. — 5 times $\frac{3}{4}$ are $\frac{15}{4}$ or $3\frac{3}{4}$; 5 times 7 are 35, which added to $3\frac{3}{4} = 38\frac{3}{4}$.

238. Multiply $3\frac{2}{5}$ by 7.
239. Multiply $8\frac{4}{7}$ by 6.
240. Multiply $10\frac{2}{9}$ by 9.
241. Multiply $11\frac{3}{11}$ by 20.

242. If one man can build $15\frac{2}{3}$ rods of wall in a week, how many rods can 13 men build in the same time?

An Integer by a Fraction.

243. Two boys shared 8 apples equally between them. What part of the whole did each receive? What is $\frac{1}{2}$ of 8 apples?

244. George and Edwin shared equally 9 apples. What part of the whole did each receive? What is $\frac{1}{2}$ of 9 apples?

245. What part of 1 inch is $\frac{1}{4}$ of 2 inches?

246. What is $\frac{1}{4}$ of 2? What is $\frac{1}{3}$ of 5? $\frac{1}{4}$ of 9?

247. What is $\frac{3}{5}$ of 25?

Solution. — $\frac{1}{5}$ of 25 is 5, and $\frac{3}{5}$ of 25 is 3 times 5, or 15. $\frac{3}{5}$ of 25 is 15.

248. What is $\frac{2}{5}$ of 15? $\frac{2}{7}$ of 14? $\frac{3}{8}$ of 32?

249. What is $\frac{5}{7}$ of 35? $\frac{7}{11}$ of 66? $\frac{11}{12}$ of 84? $\frac{9}{10}$ of 90?

250. If you do a piece of work in 16 days, in what time can you do $\frac{3}{4}$ of it?

WRITTEN EXERCISES.

251. What is the product of 72 multiplied by $\frac{3}{4}$?

$$\begin{array}{r} 4)\underline{72} \\ 18 \\ \underline{3} \\ 54 \end{array} \qquad \text{Or,} \qquad \begin{array}{r} 72 \\ \underline{3} \\ 4)\underline{216} \\ 54 \end{array}$$

Solution. — 72 × $\frac{3}{4}$ is the same as $\frac{3}{4}$ of 72. $\frac{1}{4}$ of 72 = 18, and $\frac{3}{4}$ of 72 = 3 times 18 = 54.

Or, as $\frac{3}{4}$ is the same as $\frac{1}{4}$ of 3, 72 × $\frac{3}{4}$ is the same as $\frac{1}{4}$ of 3 times 72. 3 times 72 = 216, and $\frac{1}{4}$ of 216 = 54.

252. How much is 51 multiplied by $\frac{2}{7}$?

253. What is $\frac{2}{9}$ of 101? What is $\frac{7}{8}$ of 81?

112. To multiply an integer by a fraction:

Rule.

Multiply the integer by the numerator, and divide the product by the denominator.

Multiply —

254. 48 by $\frac{2}{3}$.

255. 33 by $\frac{3}{11}$.

256. 16 by $\frac{4}{5}$.

257. 18 by $\frac{5}{6}$.

258. 51 by $\frac{2}{7}$.

259. 64 by $\frac{1}{7}$.

260. 25 by $\frac{15}{16}$.

261. 416 by $\frac{3}{8}$.

262. 23 by $\frac{5}{12}$.

263. Find the product of 101 multiplied by $\frac{2}{9}$.

264. What will 160 bushels of oats cost, at $\frac{5}{8}$ of a dollar a bushel?

265. If an acre of land is worth $239, what are $\frac{2}{11}$ of it worth?

266. What will 144 yards of cloth cost, at $\frac{9}{16}$ of a dollar per yard?

113. When the multiplier is a mixed number, *the integer may be multiplied by the fraction and the integer of the mixed number, separately, and the results added.*

267. What is the product of 33 by $3\frac{2}{5}$?

Solution.

$$\begin{array}{lr} 33 \times \frac{2}{5} = \frac{33 \times 2}{5} = \frac{66}{5} = & 13\frac{1}{5} \\ 33 \times 3 = & 99 \\ \hline 33 \times 3\frac{2}{5} = & 112\frac{1}{5} \end{array}$$

$$\begin{array}{lr} & 33 \\ & 3\frac{2}{5} \\ \hline & 5)\ 66 \\ \hline \text{Or,} & 13\frac{1}{5} \\ & 99 \\ \hline \text{Ans.} & 112\frac{1}{5} \end{array}$$

What is the product of —

268. 12 by $9\frac{3}{5}$?

269. 43 by $8\frac{5}{6}$?

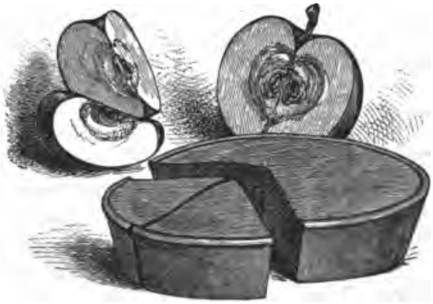
270. 67 by $5\frac{2}{13}$?

271. 112 by $12\frac{3}{8}$?

272. If a car-load of coal is worth $45, how much are $10\frac{4}{17}$ car-loads worth?

A Fraction by a Fraction.

273. If $\frac{1}{2}$ of an apple is cut into 2 equal parts, what part of the apple will 1 of the equal parts be?



274. If $\frac{1}{3}$ of a cake is cut into 2 equal parts, what part of the cake will 1 of the equal parts be?

275. What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{1}{7}$?

276. If a boy having $\frac{3}{4}$ of a melon should divide it into 4 equal parts, what part of the melon will 1 of the equal parts be?

277. What is $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{1}{4}$ of $\frac{4}{5}$? $\frac{1}{7}$ of $\frac{5}{7}$?

278. What is $\frac{2}{3}$ of $\frac{7}{9}$?

Solution.—$\frac{1}{3}$ of $\frac{7}{9}$ is $\frac{7}{27}$, and $\frac{2}{3}$ of $\frac{7}{9}$ is 2 times $\frac{7}{27}$, or $\frac{14}{27}$.

279. A boy having $\frac{3}{4}$ of a bushel of chestnuts, sold $\frac{4}{5}$ of them. What part of a bushel did he sell?

280. A merchant having $\frac{5}{12}$ of a ship sold $\frac{2}{7}$ of his part. What part of the ship did he sell?

281. What is $\frac{4}{5}$ of $\frac{3}{4}$? $\frac{2}{7}$ of $\frac{5}{12}$? $\frac{5}{9}$ of $\frac{8}{11}$?

282. What will $2\frac{1}{4}$ bushels of potatoes cost at \$$\frac{3}{4}$ a bushel?

WRITTEN EXERCISES.

283. How much is $\frac{9}{10}$ multiplied by $\frac{5}{6}$?

$$\frac{9}{10} \times \frac{5}{6} = \frac{45}{60} = \frac{3}{4}.$$

Or,

$$\frac{\overset{3}{\cancel{9}} \times \cancel{5}}{\underset{2}{\cancel{10}} \times \underset{2}{\cancel{6}}} = \frac{3}{4}.$$

Solution. — $\frac{9}{10} \times \frac{5}{6}$ is the same as $\frac{5}{6}$ of $\frac{9}{10}$. $\frac{1}{6}$ of $\frac{9}{10}$ is $\frac{9}{60}$, and $\frac{5}{6}$ of $\frac{9}{10}$ is 5 times $\frac{9}{60} = \frac{45}{60}$, $\frac{3}{4}$. Or we may shorten the process, by striking out, before multiplying, from both numerator and denominator the equal factors 3 and 5, which does not change the value of the fraction (Art. 101), and have as before $\frac{3}{4}$.

NOTE. — This striking out of equal factors, which in effect divides both terms of the fractions by the same number, is called *cancellation.*

284. How much is $\frac{8}{11}$ of $\frac{5}{12}$?

285. Multiply $\frac{7}{15}$ by $\frac{4}{5}$.

114. To multiply a fraction by a fraction:

Rule.

Multiply the numerators together for a new numerator and the denominators together for a new denominator.

Multiply —

286. $\frac{8}{11}$ by $\frac{1}{8}$.

287. $\frac{4}{5}$ by $\frac{5}{6}$.

288. $\frac{11}{12}$ by $\frac{6}{7}$.

289. $\frac{5}{3}$ by $\frac{2}{5}$.

290. $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{2}$.

291. $\frac{9}{15} \times \frac{4}{3}$ by $\frac{2}{5}$.

292. $\frac{11}{18}$ by $\frac{7}{4} \times \frac{2}{11}$.

293. $\frac{7}{8}$ by $\frac{1}{5}$ of $\frac{8}{7}$.

294. What is the product of $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{4}{5}$?

295. What part of a ship is $\frac{1}{4}$ of $\frac{5}{8}$ of it?

296. What will $\frac{5}{16}$ of a yard of cloth cost at $\$\frac{8}{25}$ a yard?

115. When among the factors there are mixed numbers, *reduce them to improper fractions and then multiply.*

297. What is the product of $4\frac{3}{5}$ by $6\frac{2}{3}$?

$$4\frac{3}{5} = \frac{23}{5}, \text{ and } 6\frac{2}{3} = \frac{20}{3},$$

$$\frac{23}{\cancel{5}} \times \frac{\overset{4}{\cancel{20}}}{3} = \frac{92}{3} = 30\frac{2}{3}.$$

Solution. — Reducing the mixed numbers to improper fractions, we have $\frac{23}{5}$ and $\frac{20}{3}$. $\frac{23}{5} \times \frac{20}{3}$ gives $\frac{92}{3} = 30\frac{2}{3}$.

298. What is the product of $7\frac{1}{2}$ by $9\frac{1}{4}$?

299. What will $5\frac{1}{4}$ pounds of sugar cost, at \$ $\frac{1}{6}$ a pound?

300. When cloth is \$ $\frac{7}{8}$ a yard, what will $\frac{4}{7}$ of 12 yards cost?

301. What is the value of $\frac{3}{4}$ of $\frac{7}{8}$ of 9?

302. What will 50 $\frac{5}{16}$ pounds of beef cost, at \$ $\frac{4}{25}$ a pound?

DIVISION OF FRACTIONS.

A Fraction by an Integer.

303. If $\frac{2}{5}$ of an apple is divided between two boys, what part of the apple will each receive?

304. Jane had $\frac{6}{7}$ of a melon, and divided the same among 3 playmates. What part of the melon did each receive?

305. What part of 1 is $\frac{1}{2}$ of $\frac{2}{5}$? $\frac{1}{3}$ of $\frac{6}{7}$?

306. How much is $\frac{9}{14}$ divided by 3? $\frac{81}{95}$ divided by 9.

307. How much is $\frac{7}{8}$ divided by 5?

Solution.—$\frac{7}{8}$ divided by 5 is the same as $\frac{1}{5}$ of $\frac{7}{8}$, and $\frac{1}{5}$ of $\frac{7}{8}$ is $\frac{7}{40}$.

308. Divide $\frac{2}{7}$ by 3; $\frac{7}{9}$ by 5; $\frac{3}{10}$ by 8; $\frac{8}{11}$ by 3.

309. If 5 boys have $\frac{11}{12}$ of a melon divided among them, what part of a melon will each receive?

310. If 1 man does a piece of work in $\frac{9}{10}$ of a day, in what time can 6 men do it?

311. If 5 bushels of apples cost \$ $6\frac{1}{4}$, what is the cost of 1 bushel?

116. Principle.

A fraction is divided either by dividing its numerator or by multiplying its denominator.

WRITTEN EXERCISES.

312. What is the quotient of $\frac{18}{23}$ divided by 6?

$$\tfrac{18}{23} \div 6 = \tfrac{18 \div 6}{23} = \tfrac{3}{23}.$$

Or,

$$\tfrac{18}{23} \div 6 = \tfrac{18}{23 \times 6} = \tfrac{18}{138} = \tfrac{3}{23}.$$

Solution. — $\frac{18}{23} \div 6$ is the same as $\frac{1}{6}$ of $\frac{18}{23}$, and $\frac{1}{6}$ of $\frac{18}{23}$ is $\frac{3}{23}$.

Or, $\frac{18}{23} \div 6 = \frac{18}{23 \times 6} = \frac{18}{138} = \frac{3}{23}$.

313. Divide $\frac{42}{45}$ by 14.

314. Divide $\frac{7}{9}$ by 8.

117. To divide a fraction by an integer:

Rule.

Either divide the numerator or multiply the denominator by the integer.

Divide —

315. $\frac{7}{8}$ by 4.

316. $\frac{21}{25}$ by 7.

317. $\frac{48}{37}$ by 12.

318. $\frac{7}{40}$ by 21.

319. $\frac{18}{20}$ by 27.

320. $\frac{24}{47}$ by 16.

321. $\frac{36}{65}$ by 18.

322. $\frac{45}{63}$ by 25.

323. $\frac{15}{16}$ by 6.

324. When 11 pounds of coffee can be bought for $\frac{33}{16}$ of a dollar, how much is it a pound?

118. When the dividend is a mixed number, *it must be reduced to an improper fraction, before dividing.*

325. Divide $14\frac{3}{4}$ by 8.

326. Divide $21\frac{3}{7}$ by 25.

327. Divide $4\frac{1}{13}$ by 9.

328. Divide $30\frac{1}{4}$ by 22.

329. How much is $\frac{5}{8}$ of \$11 divided by 22.

330. At \$15 a ton, how many tons of hay can be bought for \$$116\frac{1}{4}$?

331. When 4 bushels of peaches cost \$$4\frac{1}{2}$, what is the cost of 1 bushel?

332. If \$$28\frac{4}{5}$ be divided equally among 12 men, how much will each receive?

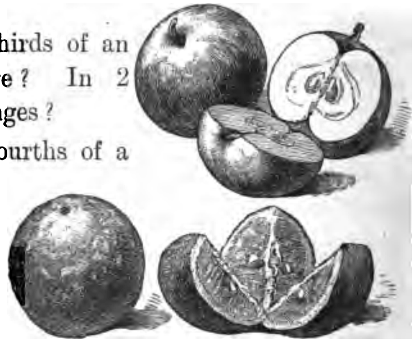
An Integer by a Fraction.

333. How many halves of an apple in 1 apple? In 2 apples?

334. How many thirds of an orange in 1 orange? In 2 oranges? In 3 oranges?

335. How many fourths of a dollar in \$1? In \$3? In \$5?

336. How many times $\frac{2}{3}$ in 5?



Solution. — $5 = \frac{15}{3}$; and $\frac{2}{3}$ are contained in $\frac{15}{3}$ as many times as 2 in 15, or $7\frac{1}{2}$ times.

337. At \$$\frac{3}{4}$ a bushel, how many bushels of corn can be bought for \$4?

338. At \$$\frac{5}{6}$ each, how many books can be bought for \$10?

339. How many bags containing $1\frac{1}{2}$ bushels can be filled from a bin containing 9 bushels?

WRITTEN EXERCISES.

340. How many times $\frac{2}{5}$ in 16?

$$16 = \frac{80}{5},\ \frac{80}{5} \div \frac{2}{5} = 80 \div 2 = 40.$$

Or,

$$16 \div \frac{2}{5} = \frac{16 \times 5}{2} = 40.$$

Solution. — $16 = \frac{80}{5}$, and $\frac{2}{5}$ are contained in $\frac{80}{5}$ as many times as 2 in 80, or 40 times.

Or, $\frac{1}{5}$ is contained in 16, 5 times 16, or 80 times; and $\frac{2}{5}$, one half of 80 times, or 40 times.

341. How many times $\frac{6}{7}$ in 19?

342. How many times $\frac{5}{8}$ in 21?

119. To divide an integer by a fraction:

Rule.

Change the integer to a fraction having the same denominator as the divisor, and then divide the numerator of the dividend by the numerator of the divisor. Or,

Multiply the integer by the denominator of the divisor and divide the product by the numerator.

Divide —

343. 8 by $\frac{4}{5}$.

344. 14 by $\frac{24}{25}$.

345. 3 by $\frac{9}{10}$.

346. 15 by $\frac{10}{11}$.

347. 75 by $\frac{3}{4}$.

348. 5 by $\frac{7}{10}$.

349. 27 by $\frac{5}{9}$.

350. 34 by $\frac{17}{20}$.

351. 12 by $\frac{8}{16}$.

352. If it takes $\frac{2}{3}$ of a yard of cloth to make a vest, how many vests can be made from 6 yards?

120. When the divisor is a mixed number, *change it to an improper fraction, before dividing.*

353. Divide 75 by $3\frac{2}{3}$.

355. Divide 92 by $5\frac{1}{9}$.

354. Divide 18 by $1\frac{1}{4}$.

356. Divide 100 by $4\frac{7}{8}$.

357. At $6\frac{1}{4}$ cents a pound, how many pounds of rice can be bought for $1.50 ?

358. How many bushels of wheat, at $ $2\frac{1}{6}$ a bushel, can be bought for $ 39 ?

A Fraction by a Fraction.

359. How many times $\frac{1}{5}$ of a melon in $\frac{2}{5}$ of a melon ? In $\frac{4}{5}$ of a melon ?

360. At $ $\frac{1}{4}$ a yard, how many yards of cloth can be bought for $ $\frac{3}{4}$?

361. At $ $\frac{2}{5}$ a gallon, how many gallons of vinegar can be bought for $ $\frac{4}{5}$?

362. At $ $\frac{3}{16}$ a yard, how many yards of cloth can be bought for $ $\frac{3}{4}$? How many times $\frac{3}{16}$ in $\frac{3}{4}$?

Solution. — $\frac{3}{4}$ are $\frac{12}{16}$; $\frac{3}{16}$ is contained in $\frac{12}{16}$, as many times as 3 in 12, or 4 times.

363. At $ $\frac{3}{10}$ each, how many slates can be bought for $ $\frac{3}{5}$?

364. How many times $\frac{2}{14}$ in $\frac{2}{7}$? $\frac{3}{10}$ in $\frac{9}{10}$?

365. How many times $\frac{3}{8}$ in $\frac{12}{16}$? $\frac{2}{9}$ in $\frac{17}{18}$?

366. At $ $1\frac{1}{2}$ a bushel, how many bushels of wheat can be bought with $ $4\frac{1}{2}$?

WRITTEN EXERCISES.

367. What is the quotient of $\frac{4}{5}$ divided by $\frac{2}{3}$.

$$\frac{4}{5} \div \frac{2}{3} = \frac{12}{15} \div \frac{10}{15} = \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$$

Or,

$$\frac{4}{5} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{5} \times \frac{3}{\cancel{2}} = \frac{6}{5} = 1\frac{1}{5}.$$

Solution. — $\frac{4}{5} \div \frac{2}{3}$ is equivalent to $\frac{12}{15} \div \frac{10}{15}$, which is the same as $\frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$. Or, Since $\frac{4}{5} \div 1$ is $\frac{4}{5}$, $\frac{4}{5} \div \frac{1}{3}$ is $\frac{4 \times 3}{5}$, and $\frac{4}{5} \div \frac{2}{3}$ is $\frac{4 \times 3}{5 \times 2}$, or $\frac{4}{5} \times \frac{3}{2}$, which gives $1\frac{1}{5}$, the same result as before. This is the same as multiplying the dividend by the divisor inverted.

368. How many times $\frac{3}{7}$ in $\frac{1}{12}$?

369. How many times $\frac{5}{9}$ in $\frac{13}{15}$?

121. To divide one fraction by another:

Rule.

Change the fractions, if necessary, to fractions having a common denominator, and divide the numerator of the dividend by the numerator of the divisor. Or,

Invert the divisor and proceed as in multiplication of fractions.

NOTE. — If there are mixed numbers change them to improper fractions.

Divide —

370. $\frac{7}{8}$ by $\frac{1}{2}$.
371. $\frac{2}{7}$ by $\frac{4}{11}$.
372. $\frac{5}{6}$ by $\frac{2}{9}$.
373. $\frac{8}{9}$ by $\frac{4}{7}$.
374. $\frac{9}{13}$ by $\frac{3}{4}$.
375. $\frac{9}{30}$ by $4\frac{1}{2}$.
376. $3\frac{1}{6}$ by $9\frac{1}{2}$.
377. $\frac{7}{16}$ by $\frac{7}{8}$.
378. $9\frac{1}{2}$ by $\frac{7}{8}$.

379. How many pounds of tea, at \$$\frac{1}{4}$ a pound, can be bought for \$$7\frac{1}{2}$?

380. When for $ $\frac{7}{8}$ $3\frac{1}{2}$ bushels of apples can be bought, what is the cost of 1 bushel?

381. At $ $\frac{7}{9}$ for $\frac{2}{5}$ of a yard of cloth, what is the price per yard?

382. At $ $\frac{5}{8}$ a gallon, how many gallons of kerosene can be bought for $ $5\frac{1}{2}$?

QUESTIONS.

85. What is a fraction? 86. What is the denominator? 87. The numerator?

88. What is a common fraction? 89. How may an integer be expressed in a fractional form?

90. What is a mixed number? 91. What may a fraction be regarded? 92. What is the value of a fraction?

93. What is a proper fraction? 94. An improper fraction?

95. What is reduction of fractions? 96. How are integers or mixed numbers reduced to fractions? 97. How is an improper fraction reduced to an integer or mixed number?

98. What is a common divisor of two or more numbers? 99. What are the terms of a fraction? 100. When is a fraction in its smallest terms? 102. How is a fraction reduced to its smallest terms?

103. When have fractions a common denominator? 104. How are fractions reduced to fractions having a common denominator?

105. How are fractions added? 107. How is one fraction subtracted from another?

109. What is a principle in multiplying fractions? 110. How is a fraction multiplied by an integer? 112. An integer by a fraction? 114. A fraction by a fraction?

116. What is a principle in division of fractions? 117. How is a fraction divided by an integer? 119. An integer by a fraction? 121. A fraction by a fraction?

REVIEW.

122. 1. How many fifths in $6\frac{4}{5}$? In $7\frac{3}{5}$?

2. A horse can trot $\frac{45}{7}$ of a mile in an hour; how-many miles is that?

3. In 15 weeks a certain school has had 31 half-days vacation; how many days is that? How many weeks?

4. In $ $4\frac{3}{5}$ how many fifths of a dollar?

5. Jane has $ $\frac{3}{4}$, Susan $ $\frac{5}{8}$ and Albert $ $\frac{1}{2}$. How much have they in all?

6. If you should have $\frac{9}{16}$ of a bushel of nuts and should sell $\frac{3}{8}$ of a bushel, what part of a bushel would you have left?

7. What is the difference between $4\frac{1}{10}$ and $3\frac{1}{5}$?

8. At $ $\frac{3}{4}$ a pound, how many dollars will pay for 7 pounds of tea?

9. At $ $8\frac{1}{5}$ a barrel, what will cost $\frac{1}{4}$ of a barrel of flour?

10. At $\frac{2}{3}$ of a dollar a yard, how much must be paid for $\frac{3}{8}$ of a yard of cloth?

11. 12 is $\frac{3}{4}$ of what number?

Solution. — As 12 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 12, or 4; as 4 is $\frac{1}{4}$ of a number $\frac{4}{4}$ of it must be 4 times 4, or 16.

12. 24 is $\frac{4}{5}$ of what number?

13. 40 is $\frac{8}{9}$ of what number?

14. At $ $\frac{5}{6}$ a bushel, what part of a bushel of corn can be bought for $ $\frac{3}{4}$?

15. If a man can mow $1\frac{1}{2}$ acres of grass in one day, how many acres can he mow in $10\frac{1}{2}$ days?

16. Sold a watch for $ 38, which was $\frac{8}{7}$ of what it cost. How much was gained by the sale?

17. 28 is $\frac{7}{10}$ of how many times 9?

Solution. — 28 is $\frac{7}{10}$ of 40, and 40 is $4\frac{4}{9}$ times 9.

18. 32 is $\frac{4}{7}$ of how many times 6?

19. 48 is $\frac{4}{5}$ of how many times 10?

20. 36 is $\frac{3}{4}$ of how many times 8?

21. $\frac{5}{7}$ of 21 is how many fifths of 75?

22. $\frac{5}{6}$ of 48 is how many thirds of 60?

23. Bought 12 yards of cloth and sold it for $ 54, which was $\frac{9}{8}$ of its cost? What was its cost a yard?

24. David is 9 years old, and $\frac{2}{3}$ of his age is exactly $\frac{3}{4}$ of the age of his brother. What is the age of his brother?

WRITTEN EXERCISES.

25. Reduce $\frac{84}{132}$ to its smallest terms.

26. Reduce $\frac{144}{240}$ to its smallest terms.

27. Change 63 to a fractional form.

28. Change $42\frac{3}{7}$ to an improper fraction.

29. Change 31 to a fraction whose denominator is 3.

30. Change $\frac{1}{4}$ and $\frac{1}{5}$ to fractions having a common denominator.

31. Add $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{9}$.

32. What is the value of $\frac{5}{8} + \frac{5}{6} + \frac{2}{3}$?

33. Find the sum of $\frac{3}{5}$, $2\frac{3}{10}$, and $5\frac{1}{4}$.

34. How much larger is $\frac{8}{15}$ than $\frac{9}{20}$?

35. Find the difference between $\frac{10}{11}$ and $\frac{6}{7}$.

36. From $6\frac{11}{14}$ subtract $2\frac{11}{21}$.

37. Change $\frac{131}{5}$ to a mixed number.

38. Find the value of $\frac{13}{14} \times 6$.

39. Multiply 51 by $\frac{5}{17}$.

40. How much is $\frac{8}{9}$ of $\frac{1}{2}$ of $\frac{6}{7}$?

41. Find the value of $\frac{3}{11} \times \frac{7}{4} \times \frac{8}{3}$.

42. Find the value of $(\frac{17}{20} \div \frac{4}{15}) \times \frac{13}{16} - \frac{5}{8}$.

43. Find the value of $(\frac{2}{9} + \frac{5}{4} - 1\frac{1}{12}) \times 12$.

44. What is the quotient of $\frac{17}{20} \div 9$?

45. What is the quotient of $26 \div \frac{13}{15}$?

46. At $9 a barrel, what will $\frac{3}{4}$ of a barrel cost?

47. At $5.25 what will $\frac{4}{5}$ of a ton of coal cost?

48. How much is $\frac{7}{11}$ of $8.80?

49. A merchant owning $\frac{3}{8}$ of a ship bought $\frac{5}{16}$ more; how much did he then own of it?

50. A farmer divided some hay into 20 equal parts, and gave 16 parts to his cows, and the rest to his sheep; what fraction of the whole did the sheep get?

51. What number is that to which if you add $3\frac{5}{9}$, the sum will be $16\frac{5}{12}$?

52. If $\frac{7}{8}$ of a yard of cloth cost $3.78, what will a whole yard cost?

53. Paid $156 for a horse, which was $\frac{18}{15}$ as much as I paid for a wagon. How much did I pay for the wagon?

54. If $\frac{3}{8}$ of a mill is worth $3740, what is the value of the whole?

55. When 13 tons of coal are sold for 71\frac{1}{2}$, what is the price per ton?

56. Paid 10\frac{1}{2}$ for tea, at the rate of $$\frac{7}{10}$ a pound; how many pounds did I buy?

57. If the divisor is $\frac{5}{13}$, and the quotient 2$\frac{1}{4}$, what is the dividend?

58. A farmer sold 31$\frac{15}{16}$ bushels of grain, which was $\frac{7}{8}$ of all he had. How much had he?

59. If 11$\frac{11}{32}$ acres is $\frac{5}{7}$ of a certain field, what is its entire extent?

60. When $\frac{3}{4}$ of a pound of tea cost 54 cents, what is the cost of 1 pound? Of 4$\frac{5}{8}$ pounds?

61. If 9 bushels of wheat are worth $19.35, what is 1 bushel worth? What are 10$\frac{3}{4}$ bushels worth?

REVIEW QUESTIONS.

73. What is an integer? 85. What is a fraction? 99. The terms of a fraction? 92. The value of a fraction?

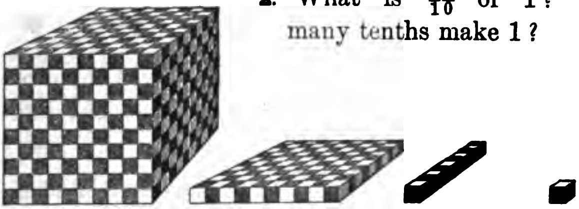
103. When have fractions a common denominator? 105. How are fractions added? 107. How is one fraction subtracted from another?

114. How is a fraction multiplied by a fraction? 121. How is one fraction divided by another.

DECIMAL FRACTIONS.

123. 1. If a block is divided into 10 equal parts, what is each of these parts called?

2. What is $\frac{1}{10}$ of 1? How many tenths make 1?



A Unit or One. **One Tenth.** **One Hundredth.** **One Thousandth.**

3. If $\frac{1}{10}$ of a block is divided into 10 equal parts, what is each of these parts called?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$? How many hundredths make 1?

5. If $\frac{1}{100}$ of a unit is divided into 10 equal parts, what is each of these parts called?

6. What is $\frac{1}{10}$ of $\frac{1}{100}$? How many thousandths make $\frac{1}{10}$?

124. A **Decimal Fraction** is a fraction whose unit is divided into tenths, hundredths, thousandths, etc.

When written without its denominator, by means of the decimal point, it is called a *Decimal* (Art. 23).

The *orders of decimals* as named from the decimal point toward the right are shown in the following

TABLE.

ORDERS.	Thousands.	Hundreds.	Tens.	Ones.		Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.
NUMBER.	6	7	8	2	.	5	0	9	1	3	0

This number is read six thousand seven hundred eighty-two, *and* five hundred nine thousand one hundred thirty millionths.

NOTE. — In reading an integer and decimal together use the word *and* only to indicate the decimal point.

Thus 507.0425 is read five hundred seven *and* four hundred twenty-five ten-thousandths, not five hundred and seven and four hundred and twenty-five ten-thousandths.

We are thus enabled to distinguish readily between .404 and 400.004, etc.

125. The **Denominator** of a decimal is 1 with as many ciphers annexed as there are orders of the decimal.

WRITTEN EXERCISES.

Write and read —

7. 0.15
8. 0.05
9. 0.006
10. 0.175
11. 0.3
12. 0.77
13. 0.608.
14. 0.1325
15. 0.506
16. 0.7153
17. 0.0174
18. 0.006501
19. 0.4026
20. 0.006
21. 15.15
22. 35.1675
23. 91.289
24. 102.05
25. 5067.5
26. 183.57
27. 4.300068.

Write in the decimal form —

28. $\frac{7}{10}$.

29. $\frac{3}{100}$.

30. $\frac{5}{1000}$.

31. $\frac{65}{1000}$.

32. $\frac{106}{1000}$.

33. $\frac{7}{1000}$.

34. $\frac{14}{10000}$.

35. $\frac{1068}{10000}$.

36. $7\frac{16}{100}$

37. $19\frac{9}{1000}$.

38. $103\frac{21}{100}$.

39. $162\frac{121}{10000}$.

40. Three hundred six thousandths.

41. Twenty-three ten-thousandths.

42. Thirty-six, and five tenths.

43. One hundred one, and sixty-five hundredths.

44. Sixteen and sixteen thousandths.

45. Three hundred twenty-five ten-thousandths.

46. Five and three hundred thousandths.

47. Nineteen and six hundred ten-thousandths.

REDUCTION OF DECIMALS.

Decimals to Lower or Higher Orders.

48. In 1 dollar how many tenths of a dollar? How many hundredths? How many thousandths?

49. In 6 tenths how many hundredths? In 7?

50. In 60 hundredths how many tenths? In .50?

51. In 5 how many thousandths? In 9?

52. In 5000 how many ones? In 9000?

53. In .03 how many thousandths? In .06? In .11?

54. In .030 how many hundredths? In. .060?

126. Principle.

Annexing ciphers to a decimal, or removing ciphers from the right of a decimal, does not change its value.

WRITTEN EXERCISES.

55. Change .151 to hundred-thousandths.

$.151 = .15100$

Solution.—.151 is changed to ten-thousandths by annexing one cipher, and the ten-thousandths to hundred-thousandths by annexing a second cipher. Ans. .15100.

56. Change .6 and .085 to decimals having the same denominator.

57. Change .06 and .1346 to decimals having a common denominator.

127. To reduce decimals to equivalent decimals having a common denominator:

Rule.

Give each decimal the same number of decimal orders, by annexing ciphers.

Change to decimals having a common denominator—

58. .7 and .05

59. .06 and .183.

60. .5216 and .16.

61. .0031 and .316.

62. .161 and .01.

63. .8 and .09163.

A Decimal to a Common Fraction.

64. What is the denominator of .5? What is .5 expressed in the form of a common fraction?

65. How many halves in $\frac{5}{10}$? Fifths in $\frac{6}{10}$? In .6? In .8?

66. How many fourths in $\frac{25}{100}$? In .25? In .75?

WRITTEN EXERCISES.

67. Change .35 to a common fraction in smallest terms.

$$.35 = \frac{35}{100} = \frac{7}{20}$$

Solution.—As .35 is 35 hundredths it is equal to $\frac{35}{100}$, which reduced to its smallest terms is $\frac{7}{20}$.

68. Change .125 to a common fraction in smallest terms.

69. Change .064 to a common fraction in smallest terms.

128. To reduce a decimal to a common fraction:

Rule.

Omit the decimal point, write the denominator, and then reduce the fraction to its smallest terms.

Change to common fractions—

70. .375

71. .75

72. .06

73. .6875

74. .0075

75. .0625

76. .0075

77. .216

78. 5.44

79. 4.008

80. 1.025

81. 2.4375

A Common Fraction to a Decimal.

82. How many tenths in 1? In $\frac{1}{2}$? In $\frac{1}{5}$?

83. How many hundredths in 1? In $\frac{1}{4}$? In $\frac{1}{2}$? In $\frac{3}{4}$?

84. In a dime how many tenths? In a dollar how many hundredths? In half a dollar how many hundredths?

WRITTEN EXERCISES.

85. Change $\frac{7}{8}$ to a decimal.

$$\begin{array}{r@{\,}l} 8) & \underline{7.000} \\ & .875 \end{array}$$

Solution. — 7 ones are 70 tenths; $\frac{1}{8}$ of 70 tenths is 8 tenths, and a remainder of 6 tenths; 6 tenths are 60 hundredths; $\frac{1}{8}$ of 60 hundredths is 7 hundredths, and a remainder of 4 hundredths; 4 hundredths are 40 thousandths; $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Ans. .875.

86. Change $\frac{7}{20}$ to a decimal.

87. Reduce $\frac{13}{25}$ to a decimal.

88. Reduce $6\frac{3}{4}$ to a mixed decimal.

89. Reduce $3\frac{4}{5}$ to a mixed decimal.

129. To reduce a common fraction to a decimal:

Rule.

Annex to the numerator as many decimal ciphers as may be required, and then divide by the denominator.

NOTE. — The sign + may be used in the quotient, after enough decimal places have been found, to show that the division is incomplete. Thus, $\frac{5}{12}$, reduced to four decimal places, is equal to $5.0000 \div 12 = .4166+$.

Reduce to decimals —

90. $\frac{5}{8}$.
91. $\frac{1}{25}$.
92. $\frac{3}{8}$.
93. $\frac{17}{20}$.
94. $\frac{3}{125}$.
95. $\frac{15}{16}$.
96. $\frac{8}{25}$.
97. $\frac{3}{5}$.
98. $\frac{1}{40}$.
99. $\frac{12}{125}$.
100. $\frac{6}{25}$.
101. $\frac{63}{24}$.
102. $\frac{14}{5}$.
103. $2\frac{5}{8}$.
104. $5\frac{1}{25}$.

105. Reduce $\frac{4}{9}$ to a decimal of three places.

106. Reduce $\frac{6}{7}$ to a decimal of four places.

107. Reduce $\frac{5}{12}$ to a decimal of four places.

108. Reduce $\frac{7}{75}$ to a decimal of four places.

ADDITION OF DECIMALS.

109. What is the sum of $\frac{5}{10}$ and $\frac{3}{10}$? Of .4 and .5?

110. What is the sum of $\frac{11}{100}$ and $\frac{15}{100}$?

111. What is the sum of 6, 0.5, and 0.8?

112. What is the sum of $\frac{25}{100}$ and $\frac{5}{1000}$?

WRITTEN EXERCISES.

113. Find the sum of 19.03, 6.5, and .19.

$$\begin{array}{r} 19.03 \\ 6.5 \\ .19 \\ \hline 25.72 \end{array}$$

Solution. — We write figures of the same order in the same column, and as ten units of any order are one of the next higher order, add as in integers. The sum found is 25.72.

	114.	**115.**	**116.**
Add	2.35	41.144	.0632
	42.08	13.755	.456
	9.52	2.50	.81
	18.005	25.3	3.08

117. Add 6.051, .095, 31.6, and 1.0075.

118. Add 13.015, .9031, 12.186, and .09.

119. Add 120.16, .003, 1401.7, 1.5, and 19.8.

120. Add 7 and 8 thousandths, 28 and 16 hundredths, 56 and 7 tenths.

121. A merchant bought four lots of goods; for the first he paid $ 69.125; for the second, $ 193.30; for the third, $ 1008.56; and for the fourth, $ 752.375; how much did he pay for the whole?

SUBTRACTION OF DECIMALS.

122. What is the difference between $\frac{8}{10}$ and $\frac{3}{10}$?

123. What is the difference between $\frac{31}{100}$ and $\frac{25}{100}$?

124. What is .6 less .07? .034 less .029?

125. What is the difference between 3 and .3?

WRITTEN EXERCISES.

126. Find the difference between 18.96 and 9.375.

$$\begin{array}{r} 18.960 \\ \underline{9.375} \\ 9.585 \end{array}$$

Solution. — We write the subtrahend under the minuend, so that figures of the same order are in the same column. As there are no thousandths in the minuend, we fill the order with a 0. Then, since ten units of any order are one of the next higher order, we subtract as in integers. The difference found is 9.585.

	127.	**128.**	**129.**
From	17.083	97.031	106.570
take	9.36	18.346	41.139

130. From $ 37 take $ 31.375.

131. Bought 160 acres of land, and sold from it 141.125 acres. How much had I left?

132. From 15 take 15 hundredths.

133. A farmer, owning 78 hundredths of a farm, sold 725 thousandths of it. How much of it had he left?

134. From 10 take 1 ten-thousandth.

135. Subtract 139.216 from 400.95.

136. What is the difference between 64.075 and .195326?

137. From 107 take .0007.

138. From forty-three, and seventy-five thousandths, take thirty-five, and sixty-seven hundred-thousandths.

MULTIPLICATION OF DECIMALS.

139. What is 2 times $\frac{3}{10}$? 4 times .2? 5 times .3?

140. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{3}{10}$? .1 of .5?

141. What is 3 times $\frac{2}{100}$? 2 times .03?

142. How many decimal places are there in the product of tenths by ones? In the product of tenths by tenths? In the product of hundredths by tenths?

WRITTEN EXERCISES.

143. What is the product of 3.16 multiplied by .4.

$$\begin{array}{r} 3.16 \\ .4 \\ \hline 1.264 \end{array}$$

Solution.—As .4 is the same as $\frac{1}{10}$ of 4, 3.16 multiplied by .4 is the same as $\frac{1}{10}$ of 4 times 3.16. 4 times 3.16 = 12.64, and $\frac{1}{10}$ of 12.64, which is found by removing the decimal point one place to the left (Art. 77), is 1.264. Ans. 1.264.

144. What is the product of 16.05 by 15?

145. What is the product of 136.2 by .08?

130. To multiply in decimals,

Rule.

Multiply as in integers, and point off as many decimal figures in the product as there are decimal places in both factors.

If there are not as many figures in the product as there are decimal places in both factors, supply the deficiency by prefixing ciphers.

	146.	**147.**	**148.**
Multiply	.123	3162	.7869
by	.6	.08	100

Multiply —

149. 3.21 by 2.31.

150. 1000 by .7.

151. .106 by .35.

152. 3.18 by 41.7.

153. .001 by .01.

154. 78 by .42.

155. 9.16 by 10.

156. 65 by .65.

157. .036 by .08.

158. 75 by .8.

159. .009 by .009.

160. .063 by 1000.

161. What cost .65 of an acre of land, at $84 an acre?

162. Multiply 12 hundredths by 8 thousandths.

163. What cost 18.75 barrels of flour, at $6.75 a barrel?

164. Find the value of .1 × .1 × .1.

165. Find the value of .0002 × 3.01.

166. What cost 18.25 yards of cambric, at $0.125 a yard?

167. When $14.56 is the cost of a load of coal, what is the cost of 2.25 loads?

168. Multiply thirty-six, and forty-eight hundredths, by four hundred seventy-five thousandths.

DIVISION OF DECIMALS.

169. What is $\frac{1}{2}$ of $\frac{2}{10}$? $\frac{1}{3}$ of $\frac{9}{10}$? $\frac{1}{4}$ of $\frac{12}{100}$?

170. What is .6 ÷ 2? .35 ÷ 5? .63 ÷ 7?

171. How many times $\frac{3}{10}$ in $\frac{9}{10}$? $\frac{7}{100}$ in $\frac{21}{100}$?

172. How many times .4 in .8? .06 in .42?

173. What is .54 ÷ 9? .54 ÷ .09?

174. How many decimal places in the quotient when tenths are divided by ones? Tenths by tenths? Hundredths by ones? Hundredths by hundredths?

WRITTEN EXERCISES.

175. Find the quotient of 1264 divided by .4.

$$\begin{array}{r} .4\overline{)\,1264.0} \\ \hline 3160 \end{array}$$

Solution. — 1264 = 12640 tenths or 1264.0, and 1264.0 divided by .4 gives as the quotient 3160.

176. Find the quotient of 3192 divided by .6?

177. Find the quotient of 1575 divided by .05?

178. What is the quotient of .7935 divided by .23?

```
          3.45
\23.) \79.35
        69
        ---
        103
         92
        ---
         115
         115
        ---
```

Solution. — Multiplying the divisor and dividend both by 100, by removal of the decimal point two places to the right, which will not change the quotient (Art. 101), we have for divisor 23 and for dividend 79.35, which give for the quotient 3.45.

131. To divide in decimals,

Rule.

If the divisor is a whole number, divide as in integers, and point off as many decimal places in the quotient as there are such places in the dividend. Or,

If the divisor is a decimal, make it an integer by removing the decimal point a sufficient number of places to the right, and remove the decimal point in the dividend as many places to the right, and then divide.

When the division does not terminate, or has been carried to sufficient exactness, the sign + may be annexed to the quotient to denote the incompleteness.

NOTE. — By having the quotient placed under or over the dividend, its decimal point will invariably come under or over that of the dividend.

Divide—

179. 8.05 by .23.

180. 8.05 by 2.3.

181. 8.05 by 23.

182. .805 by 2.3.

183. 1.31 by 100.

184. 1. by .13.

185. 15.625 by 2.5.

186. 164. by .04.

187. 80.5 by 2.3. **189.** 1.31 by 10.

188. 9.065 by .049. **190.** 1.31 by 1000.

191. Divide 2.117 by .0073.

192. Divide 8 by .002.

193. Divide 7.43 by .0079 to four decimal places.

194. Divide 89 by .003 to two decimal places.

195. Find the quotient of 80.3 divided by .22.

196. Find the quotient of 80.3 divided by 2.2.

197. If 3.5 acres produce 142.1 bushels of corn, what is the yield per acre?

198. How many yards of cotton cloth, at $.125 per yard, can be bought for $46.95?

199. At $0.75 a day, how many days must a boy work to earn $141?

200. If you can walk 12.5 miles in one day, how many days will it take you to walk 102.5 miles?

201. Bought 30 pounds of butter for $5.85; how much was that a pound?

202. Divide one hundredth by one ten-thousandth.

QUESTIONS.

123. What is a decimal fraction? The orders of decimals in the table? 125. The denominator of a decimal?

126. What is a principle of decimals?

127. How are decimals reduced to decimals having a common denominator? 128. How is a decimal reduced to a common fraction? 129. How is a common fraction reduced to a decimal?

130. How do you multiply in decimals? 131. How do you divide in decimals?

REVIEW.

WRITTEN EXERCISES.

1. Add seven hundred forty-three thousandths; seven hundredths; five, and sixteen ten-thousandths.

2. Add eighty-seven, and twenty-nine hundredths; seventy-three, and forty-seven thousandths; three thousand five, and one hundred six ten-thousandths; twenty-eight and three hundredths, one and five thousandths.

3. Change $5\frac{3}{16}$ to the decimal form.

4. Change $\frac{17}{25}$ and $\frac{13}{250}$ to decimals having a common denominator.

5. From 100 subtract 101 thousandths.

6. From eighteen, and three hundred sixty-five thousandths, take eighteen hundred sixty-five ten-thousandths.

7. Change .68 to an equivalent common fraction.

8. Change 5.25 to an equivalent common fraction.

9. Multiply one hundred and thirty-seven hundredths by nine tenths.

10. What common fraction is equal to the sum of .125 and .525?

11. Multiply .0235 by 8.08.

12. Divide .0625 by 2.5.

13. At $1.25 a pair, how many pairs of shoes can be bought with $45?

14. If an acre produces 16.25 bushels of wheat, how much will 3.05 acres produce ?

15. If $ 18.5625 is paid for 148.5 yards of calico, how much is paid per yard ?

16. If 3.5 cords of wood cost $ 22.4, how much will 4.25 cords cost ?

17. If 9 bushels of wheat are worth $ 19.35, what are 10.6 bushels worth ?

18. If 17 bushels of corn cost $ 16.32, what will 15.75 bushels cost ?

19. If $\frac{3}{8}$ of a cord of wood cost $ 2.85, what will .75 of a cord cost ?

20. If .75 of a pound of tea cost $ 0.54, what will $4\frac{3}{9}$ pounds cost ?

21. If 4.5 pounds of butter are worth $ 1.44, what will $11\frac{7}{10}$ pounds cost ?

REVIEW QUESTIONS.

85. What is a fraction ? 99. What are terms of a fraction ? 92. What is the value of a fraction ?

88. What is a common fraction ? 124. What is a decimal fraction ? 125. What is the denominator of a decimal ?

90. What is a mixed number ? 96. How may a mixed number be reduced to an improper fraction ? 89. How may an integer be expressed in a fractional form ?

103. When have fractions a common denominator ? 104. How may fractions be reduced to fractions having a common denominator ? 127. How may decimals be reduced to decimals having a common denominator ?

DENOMINATE NUMBERS.

COMMON WEIGHTS AND MEASURES.

132. 1. How can the length of a house be found? How can the distance between two points be found?

2. How would you express the extent of a field? How would you express the space in a room?

3. How would you tell how much water there may be in a bucket? How would you tell how much there may be of a quantity of sugar?

133. **Common Weights and Measures**, or those in general use in the United States, are given in the following tables.

MEASURES OF LENGTHS.

134. Linear or Length Measures are those used in measuring lines and distances.

TABLE.

12	inches (in.)	are	1 foot, ft.
3	feet	"	1 yard, yd.
5½	yards	"	1 rod, rd.
320	rods	"	1 mile, mi.

4. How many inches in 2 feet? In 7 feet? In 9 feet?

5. How many feet in 4 yards? In 6 yards? In 11 yards?

6. How many yards in 2 rods? In 4 rods?

7. How many yards in 18 feet? In 30 feet?

8. How many rods in 11 yards?

Solution.—As in 1 rod there are 5½ yards, or 11 half-yards, in 11 yards, or 22 half-yards, there are as many rods as 11 half-yards are times in 22 half-yards, which are 2. There are 2 rods in 11 yards.

9. How many rods in 22 yards? In 33 yards?

10. How many inches in 2 feet 6 inches? In 1 yard 2 feet?

11. How many feet in 30 inches? How many yards in 60 inches?

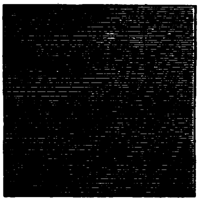
MEASURES OF SURFACES.

135. A **Square** is a plane figure having four equal straight sides and four equal corners or angles.

136. **Square Measures** are those used in measuring surfaces.

TABLE.

144 square inches (sq. in.)	are	1 square foot, sq. ft.
9 square feet	"	1 square yard, sq. yd.
30¼ square yards	"	1 square rod, sq. rd.
160 square rods	"	1 acre, A.



1 Square Inch.

12. How many square inches in 1 square foot?

13. How many square feet in 5 square yards?

14. How many square yards in 1 square rod? In 4 square rods?

15. How many square rods in 1 acre? In ½ of an acre?

16. How many square yards in 72 square feet?

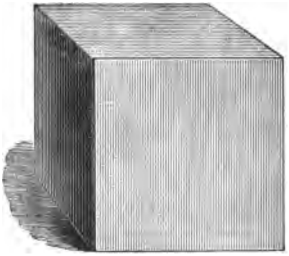
17. How many square rods in 121 square yards?

18. At $ 50 an acre what will 80 square rods of land cost?

MEASURES OF VOLUMES.

137. A **Cube** is a body or volume bounded by six equal squares.

138. **Cubic Measures** are those used in measuring bodies having length, breadth, and height or thickness.



1 Cubic Inch.

TABLE.

1728 cubic inches (cu. in.) are 1 cubic foot, cu. ft.
27 cubic feet " 1 cubic yard, cu. yd.

In measuring *wood* 16 cubic feet make 1 cord foot, and 8 cord feet (cd. ft.), or 128 cubic feet, make a cord (cd.).

19. How many cubic inches in 1 cubic foot?

20. How many cubic feet in 2 cubic yards?

21. How many cubic yards in 54 cubic feet?

22. How many cubic feet in 2 cord feet? In 4 cord feet?

23. How many cord feet in 2 cords? In 5 cords?

24. How many cord feet in 64 cubic feet? How many cords in 40 cord feet?

LIQUID MEASURES.

139. Liquid Measures are used in measuring all kinds of liquids.

TABLE.

4 gills (gi.)	are	1 pint, pt.
2 pints	"	1 quart, qt.
4 quarts	"	1 gallon, gal.

The *liquid gallon* contains 231 cubic inches. The *hogshead* regarded as a measure is 63 gallons.

25. How many gills in 4 pints? In 8 pints?

26. How many pints in 8 quarts? In 10 quarts?

27. How many quarts in 5 gallons? In 9 gallons?

28. How many gallons in 1 hogshead?

29. How many pints in 16 gills? In 32 gills?

30. How many gallons in 20 quarts? In 36 quarts?

31. How many hogsheads in 63 gallons?

32. How much must be paid for 2 gallons 2 quarts of vinegar at 35 cents per gallon?

DRY MEASURES.

140. Dry Measures are used in measuring grain, fruits, roots, etc.

TABLE.

2 pints (pt.) are 1 quart, qt.
8 quarts " 1 peck, pk.
4 pecks " 1 bushel, bu.

A *gallon*, or 4 quarts, dry measure, contains 268.8 cubic inches, and a *bushel*, 2150.4 cubic inches.

33. How many pints in 8 quarts? In 10 quarts?

34. How many quarts in 4 pecks? In 6 pecks?

35. How many pecks in 4 bushels? In 8 bushels?

36. How many quarts in 32 pints? In 40 pints?

37. How many bushels in 16 pecks? In 32 pecks?

38. What will $\frac{3}{4}$ of a bushel of chestnuts cost at 4 cents a quart?

AVOIRDUPOIS WEIGHTS.

141. Avoirdupois Weights are those used in weighing nearly all articles estimated by weight, except gold, silver, and jewels.

TABLE.

16 ounces (oz.) are 1 pound, lb.
2000 pounds " 1 ton, T.

Gold, silver, and jewels are weighed by what are called *Troy Weights*, of which 24 grains (gr.) are 1 pennyweight (pwt.), 20 pennyweights are 1 ounce (oz.), and 12 ounces are 1 pound.

A pound avoirdupois is 7000 grains troy, and a pound troy is 5760 grains.

39. How many ounces in 3 pounds avoirdupois? In 5 pounds?

40. How many pounds avoirdupois in 48 ounces? In 80 ounces?

41. How many pounds in $\frac{3}{5}$ of a ton? In $2\frac{1}{2}$ tons?

42. How much must be paid for 2 pounds 4 ounces of butter at 24 cents a pound?

43. What will 3 tons 500 pounds of hay cost at $20 a ton?

44. If 200 pounds of pork can be packed in a barrel, how many barrels will be required to pack $\frac{1}{2}$ of a ton?

TIME.

142. Time is a measured portion of duration.

TABLE.

60 seconds (sec.)	are	1 minute, min.
60 minutes	"	1 hour, h.
24 hours	"	1 day, d.
365 days	"	1 year, y.

Also, 7 days are 1 week, 12 calendar months are 1 year, and 366 days are 1 leap year.

The *calendar months* and the number of days in each are: —

1.	January	31	7.	July	31
2.	February	28 or 29	8.	August	31
3.	March	31	9.	September	30
4.	April	30	10.	October	31
5.	May	31	11.	November	30
6.	June	30	12.	December	31

In leap year February has 29 days.

45. How many seconds in 3 minutes?

46. How many minutes in 2 hours? In 3 hours?

47. How many hours in 2 days? In 3 days?

48. How many days in a common year?

49. How many days in a leap year?

50. How many minutes in 120 seconds?

51. How many minutes in 2 hours? In 4 hours?

52. How many hours in 120 minutes?

53. How many days in 48 hours? In 72 hours?

54. At $2 a day how much will a man earn in 3 weeks, omitting the Sundays?

55. In a leap year how many days are there in January and February?

56. How many days are there from noon June 25 to noon July 17?

143. A **Denomination** is the name of a unit of measure or weight.

144. A **Denominate Number** is a number composed of units of one or more denominations. Thus,

42 inches, 6 pounds, are denominate numbers.

145. A **Compound Number** is a number composed of two or more denominations, but of the same general kind. Thus,

3 feet 6 inches, 6 gallons 3 quarts, are compound numbers.

146. **Reduction** of denominate numbers is the process of changing them to others of equal value.

REDUCTION DESCENDING.

57. How many inches in 2 feet 9 inches?

Solution. — As in 1 foot there are 12 inches, in 2 feet there are 2 times 12 inches, or 24 inches; 24 inches and 9 inches are 33 inches. In 2 feet 9 inches there are 33 inches.

58. How many quarts in 8 gallons 3 quarts?

59. How many ounces in 3 lb. 6 oz. avoirdupois?

60. How many quarts in 2 bushels? In 8?

61. How many hours in 3 days 8 hours?

147. Reduction Descending is changing denominate numbers to smaller denominations.

WRITTEN EXERCISES.

62. How many feet in 12 rd. 4 yd. 2 ft.?

$$\begin{array}{l} 12\ (\text{rd.})\ 4\ \text{yd.}\ 2\ \text{ft.} \\ \underline{\ 5\tfrac{1}{2}}\ \text{yd.} \\ \ \ 6 \\ \underline{60} \\ 70\ (\text{yd.}) \\ \underline{\ \ 3}\ \text{ft.} \\ 212\ \text{ft.} \end{array}$$

Solution. — 1 rd. = $5\frac{1}{2}$ yd. and 12 rd. = 12 × $5\frac{1}{2}$ yd., or 66 yd. and the 4 yd. added make 70 yd.

1 yd. = 3 ft. and 70 yd. = 70 × 3 ft., or 210 ft., and the 2 ft. added make 212 ft. Hence, in 12 rd. 4 yd. 2 ft. there are 212 ft.

63. Reduce 15 bu. 3 pk. 6 qt. to quarts.

64. Reduce 3 d. 11 h. 45 min. to minutes.

148. Rule for Reduction Descending.

Multiply the largest denomination by the number of units it takes of the next smaller denomination to equal one of that larger, and to the product add the given number, if any, of the smaller denomination.

Multiply the sum in like manner, and so proceed until the given number is changed to units of the required denomination.

65. Reduce 1 T. 772 lb. to pounds.

66. Reduce 212 rd. 4 yd. to yards.

67. Reduce 1 lb. 3 oz. 5 pwt. to pennyweights.

68. Reduce 15 miles to feet.

69. Reduce 37 A. 132 sq. rd. to square yards.

70. In 7 cu. yd. 20 cu. ft. how many cubic inches?

71. In 4 hogsheads how many quarts?

72. Reduce 45 bu. 3 pk. 5 quarts to quarts.

73. In 10 h. 17 m. 18 s. how many seconds?

74. Reduce 84 gal. 2 qt. 1 pt. to gills.

75. How many ounces in 1250 lb.?

76. How many minutes in 3 wk. 5 d. 10 h.?

77. How many rods in 32 m. 25 rd.?

78. How many cord feet in 96 cords?

79. How many square rods in 14 A. 75 sq. rd.?

REDUCTION ASCENDING.

80. How many yards in 126 inches?

Solution. — As in 12 inches there are 1 foot, there are as many feet in 126 inches as times 12 inches in 126 inches, or 10, and 6 inches remainder. As 3 feet are 1 yard, there are as many yards in 10 feet as times 3 feet in 10 feet, or 3, and a remainder of 1 foot. In 126 inches there are 3 yards 1 foot 6 inches.

81. How many gallons in 35 quarts?

82. How many pounds of butter in 54 ounces?

83. How much must be paid for 7 pecks of potatoes at 60 cents a bushel?

84. How much must be paid for a can of milk containing 14 quarts, at 16 cents a gallon?

149. Reduction Ascending is changing denominate numbers to larger denominations.

WRITTEN EXERCISES.

85. How many rods in 212 feet?

$$\begin{array}{ll}
\text{3 ft.)} & \underline{\text{212}}\text{ ft.} \\
5\tfrac{1}{2}\text{ yd.)} & \text{70 yd.} + \text{2 ft.} \\
\underline{2} & \underline{\text{2}} \\
\text{11 hf. yd.)} & \underline{\text{140 hf. yd.}} \\
 & \text{12 rd.} + \text{8 hf. yd., or 4 yd.} \\
\text{Ans. 12 rd. 4 yd. 2 ft.} &
\end{array}$$

Solution. — As in 3 feet there is 1 yard, there are as many yards in 212 feet as times 3 feet in 212 feet, or 70, and 2 feet remainder. As in 5½ yards there is 1 rod, there are as many rods in 70 yards as times 5½ yards in 70 yards, or times 11 half yards in 140 half yards, which are 12, and 8 half yards, or 4 yards remainder.

In 212 feet there are 12 rd. 4 yd. 2 ft.

86. How many bushels in 258 quarts?

87. Change 5025 minutes to days.

150. Rule for Reduction Ascending.

Divide the given number by the number of units it takes of the same denomination to equal one of the next larger, and reserve the remainder, if any. Divide the quotient in like manner, and so proceed until the required denomination is reached.

The last quotient, and the several remainders, if any, will be the number sought.

Reduction Ascending and Reduction Descending, being reverse processes, are proofs of each other.

88. Change 2772 pounds to tons.

89. Change 1170 yards to rods.

90. Reduce 305 pennyweights to pounds.

91. Reduce 79200 feet to miles.

92. Reduce 183073 square yards to acres.

93. Reduce 361152 cubic inches to cubic yards.

94. Reduce 1008 quarts to hogsheads.

95. Reduce 1469 quarts to bushels.

96. Reduce 37038 seconds to hours.

97. In 2708 gills how many hogsheads?

98. In 20000 ounces how many pounds?

99. In 38040 minutes how many weeks?

100. In 10265 rods how many miles?

101. In 768 cord feet how many cords?

102. In 2315 square rods how many acres?

ADDITION OF COMPOUND NUMBERS.

103. Find the sum of 23 bu. 4 pk. 5 qt., 10 bu. 2 pk. 6 qt., 16 bu. 1 pk. 4 qt., and 20 bu. 1 pk. 2 qt.

23 bu.	4 pk.	5 qt.
10	2	6
16	1	4
20	1	2
71 bu.	2 pk.	1 qt.

Solution.—We write the numbers so that units of the same denomination are in the same column, and begin with the smallest denomination to add.

Adding the quarts we have 17 quarts = 2 pecks 1 quart. We write the 1 quart under the column of quarts, and add the 2 pecks with the column of pecks. The sum of the pecks is 10 pecks = 2 bushels 2 pecks. We write the 2 pecks under the column of pecks, and add the 2 bushels with the column of bushels. The sum of the bushels is 71 bushels. The sum required is 71 bu. 2 pk. 1 qt.

Write and add—

104.

50 gal.	3 qt.	1 pt.	2 gi.
25	2	1	0
46	2	1	2

105.

21 lb.	8 oz.	19 pwt.	20 gr.
10	4	11	16
9	13	0	5

106.

27 A.	71 sq. rd.
35	144
22	57
45	29

107.

5 d.	13 h.	39 min.	42 sec.
6	20	29	45
8	22	19	33
	17	59	59

108. What is the value of 225 lb. 10 oz. + 591 lb. 15 oz. + 101 lb. 2 oz. avoirdupois?

109. Two men leave the same place and travel, the one north 65 mi. 130 rd., and the other south 82 mi. 256 rd. How far were they then apart?

110. In excavating a cellar there were removed one day 16 cu. yd. 21 cu. ft. of earth, a second day 14 cu. yd. 10 cu. ft. and a third day 19 cu. ft. How much was removed in all?

SUBTRACTION OF COMPOUND NUMBERS.

111. Find the difference between 15 lb. 3 oz. 12 pwt. and 9 lb. 1 oz. 17 pwt.

15 lb.	3 oz.	12 pwt.
9	1	17
6 lb.	1 oz.	15 pwt.

Solution. — We write the subtrahend under the minuend so that units of the same denomination are in the same column, and begin with the smallest denomination to subtract.

As 17 pennyweights cannot be taken from 12 pennyweights, we increase the 12 pennyweights by 1 ounce or 20 pennyweights taken from the 3 ounces, — leaving 2 ounces, — which gives 32 pennyweights. 17 pennyweights from the 32 pennyweights leaves 15 pennyweights, which is written under the column of pennyweights.

1 ounce from 2 ounces leaves 1 ounce, which is written under the column of ounces.

9 pounds from 15 pounds leaves 6 pounds.

The difference found is 6 lb. 1 oz. 15 pwt.

Write and subtract —

112.

51 gal.	2 qt.	1 pt.
50	3	1

113.

37 bu.	2 pk.	0 qt.
19	3	7

114.	**115.**
14 T. 175 lb. 13 oz.	6 m. 60 rd. 5 yd. 0 ft.
9 163 15	75 3 1

116. From a pile of wood containing 19 cd. 86 cu. ft. there had been sold 13 cd. 96 cu. ft. How much remained?

117. A farmer raised 106 bushels of wheat, and sold 36 bu. 3 pk. 2 qt. How much had he left?

Difference between Dates.

118. A man was born May 16, 1819; how old was he April 9, 1881.

Solution.

May 16, 1819, to May 16, 1880 = 61 y.
May 16, 1880, to March 16, 1881 = 10 mo.
March 16, 1880, to April 9, 1881 = 24 d.
May 16, 1819, to April 9, 1881 = 61 y. 10 mo. 24 d.

151. To find the difference between two dates:

Find first the number of entire years, then the number of entire calendar months remaining, and then the remaining days.

119. What is the time between April 30, 1875, and March 18, 1877?

120. Required the time between the discovery of America by Columbus, Oct. 12, 1492, and the Declaration of Independence, July 4, 1776.

121. How long from the battle of Lexington, April 9, 1775, to the battle of New Orleans, Jan. 8, 1815?

MULTIPLICATION OF COMPOUND NUMBERS.

122. Find the product of 4 bu. 3 pk. 4 qt. multiplied by 4.

4 bu. 3 pk. 4 qt.
4
19 bu. 2 pk. 0 qt.

Solution.— 4 times 4 quarts are 16 quarts = 2 pecks and 0 quarts. We write the 0 under the quarts, and reserve the 2 pecks to add to the 4 times 3 pecks.

4 times 3 pecks = 12 pecks, which plus the 2 pecks reserved are 14 pecks = 3 bushels 2 pecks. We write the 2 pecks under the pecks, and reserve the 3 bushels to add to the 4 times 4 bushels.

4 times 4 bushels = 16 bushels, which plus the 3 bushels reserved are 19 bushels. We write the 19 bushels under the bushels.

The product is 19 bu. 2 pk.

Write and multiply —

123.

4 yd. 1 ft. 2 in.
12

124.

14 lb. 9 oz. 14 pwt. 17 gr.
5

125.

17 T. 1075 lb. 10 oz.
6

126.

5 d. 11 h. 15 min. 4 sec.
8

127. How many acres in an estate consisting of 5 farms, each measuring 70 A. 94 sq. rd.?

128. How much vinegar in 6 casks, each containing 49 gal. 2 qt. 1 pt.?

129. How much wood in 4 ranges, each containing 5 cords 6 cord feet?

130. If you have for labor and recreation 16 h. 47 min. 30 sec. each day, how many full days have you for such purposes in a week?

131. What is the weight of 8 loads of hay, each weighing 2 tons 1565 pounds?

DIVISION OF COMPOUND NUMBERS.

132. Find $\frac{1}{4}$ of 19 bu. 2 pk. 4 qt.

$$\begin{array}{r|l} 4) & \underline{19 \text{ bu. } 2 \text{ pk. } 4 \text{ qt.}} \\ & \;\;4 \text{ bu. } 3 \text{ pk. } 5 \text{ qt.} \end{array}$$

Solution.—$\frac{1}{4}$ of 19 bu. = 4 bu. with a remainder of 3 bu. The 3 bu. = 12 pk. which added to the 2 pk. in the dividend = 14 pk. $\frac{1}{4}$ of 14 pk. = 3 pk. with a remainder of 2 pk. The 2 pk. = 16 qt. which added to the 4 qt. in the dividend = 20 qt. $\frac{1}{4}$ of 20 qt. = 5 qt. The quotient is 4 bu. 3 pk. 5 qt.

Write and divide —

133.

9) 16 gal. 3 qt. 1 pt.

134.

4) 52 yd. 2 ft. 2 in.

135.

7) 16 T. 1546 lb. 14 oz.

136.

8) 5 d. 14 h. 20 min. 0 sec.

137. If 9 casks contain 412 gal. 3 qt. 1 pt., how much is that to a cask?

138. If 3 dozen spoons weigh 7 lb. 7 oz. 16 pwt. 9 gr., what is the weight of each dozen?

MEASUREMENT OF SURFACES.

139. How many square inches in a piece of paper 8 inches long and 1 inch wide? In a piece 8 inches long and 2 inches wide?

140. How many square feet in a path 15 feet long and 1 foot wide? In a path 15 feet long and 2 feet wide?

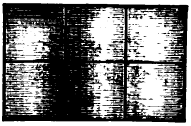
152. A **Surface** is that which has length and breadth only.

153. A **Rectangle** is a flat surface having four straight sides, and four equal angles.

A Rectangle.

154. The **Area** of a rectangle is the surface included within its sides.

The length and breadth of a rectangle determine its area.



Thus, a rectangle 3 inches long and 2 inches wide will contain in one row 3 squares of 1 square inch (Art. 136) each, and 2 such rows, or 2 times 3 square inches, make 6 square inches.

WRITTEN EXERCISES.

141. What is the area of a walk which is 25 feet long and 8 feet 6 inches wide?

$25 \times 8\frac{1}{2} = 212\frac{1}{2}$ sq. ft.

Solution.—As a walk 25 feet long and 1 foot wide will have an area of 25 square feet, a walk 8 feet 6 inches, or $8\frac{1}{2}$ feet, wide, must have an area of $8\frac{1}{2}$ times 25 square feet, or $212\frac{1}{2}$ square feet.

142. A rectangular walk has an area of 212½ square feet. Its length is 25 feet; what is its width?

212½ ÷ 25 = 8½

8½ ft. = 8 ft. 6 in.

Solution. — As the area 212½ square feet is the product of the length and width of the walk, the width must equal the quotient of the area, 212½, divided by the length, 25, which is 8½ feet = 8 feet 6 inches.

143. What is the area of a rectangular lot 31 rods long and 15 rods wide?

144. A rectangular lot has an area of 465 square rods. Its width is 15 rods; what is its length?

155. Rules for Measurement of Rectangles.

To find the area multiply the length by the breadth taken in the same denomination; and to find either side divide the area by the other side.

145. A room is 25 feet square. What is the area of its floor?

146. How many square yards of carpeting will cover a floor 18 feet long and 13 feet 6 inches wide?

147. The area of a rectangular garden is 625 square rods. One side is 25 rods; what is the other side?

148. How many acres in a rectangular farm, 64 rods long and 45 rods wide?

149. A rectangular farm containing 18 acres is 64 rods long. What is its width?

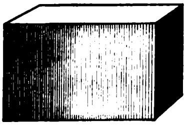
150. A hall is 60 feet long and 42 feet wide. What will it cost to cover its floor with carpeting at $ 1.25 a square yard?

MEASUREMENT OF VOLUMES.

151. How many cubic feet in a beam 10 feet long, 1 foot wide, and 1 foot deep? In a beam 10 feet long and 2 feet deep?

152. How many cubic inches in a block 8 inches long, 2 inches wide, and 1 inch thick? In a block 8 inches long, 2 inches wide, and 2 inches thick?

156. A **Volume**, or **Solid**, is that which has length, breadth, and thickness.

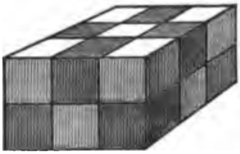


A Rectangular Volume.

157. A **Rectangular Volume** is a body bounded by six rectangles.

158. The **Contents** of a rectangular volume is the space it contains within its sides.

The length, breadth, and thickness of a rectangular volume determine its contents.



Thus, a rectangular volume 3 inches long, 3 inches wide, and 2 inches thick will contain in one layer 3 rows of 3 cubic inches, or 9 cubic inches, and 2 such layers will contain 2 times 3 times 3 cubic inches, or 18 cubic inches.

WRITTEN EXERCISES.

153. How many cubic feet in a block 8 feet long, 4 feet 3 inches wide, and 2 feet 6 inches thick?

$8 \times 4\frac{1}{4} = 34$

$34 \times 2\frac{1}{2} = 85$

Solution. — As in a block 8 feet long, 1 foot thick, and 1 foot wide, there are 8 cubic feet, there are in a block 8 feet long, 1 foot thick, and 4 feet 3 inches, or $4\frac{1}{4}$ feet, wide, $4\frac{1}{4}$ times 8 cubic feet, or 34 cubic feet, and in a block 8 feet long, $4\frac{1}{4}$ feet wide, and 2 feet 6 inches, or $2\frac{1}{2}$ feet, thick, $2\frac{1}{2}$ times 34 cubic feet, or 85 cubic feet.

154. A rectangular block contains 85 cubic feet. Its length is 8 feet, and its width 4 feet 3 inches. What is its thickness?

$$8 \times 4\tfrac{1}{4} = 34$$
$$85 \div 34 = 2\tfrac{1}{2}$$
$$2\tfrac{1}{2}\text{ ft} = 2\text{ ft. }6\text{ in.}$$

Solution. — As the contents 85 cubic feet are the product of the length, width, and thickness, the width must equal the quotient of the contents, 85, divided by the product of the length 8 multiplied by the width $4\frac{1}{4}$; or 85 divided by 34, which gives $2\frac{1}{2}$ feet = 2 feet 6 inches.

155. How many cubic feet of space in a room 16 feet long, 14 feet wide, and 10 feet high?

156. A room contains 2240 cubic feet of space. It is 16 feet long and 10 feet high. What is its width?

159. Rule for Measurement of Rectangular Volumes.

To find the contents multiply together the length, breadth, and thickness taken in the same denominations; and

To find one of the dimensions divide the contents by the product of the other two dimensions.

157. I wish to excavate a cellar 42 feet long, 21 feet wide, and 9 feet deep. How many cubic feet of earth will need to be removed?

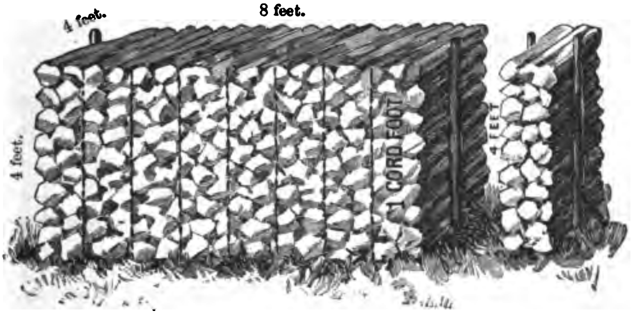
158. A wall is 36 feet long, 2 feet 6 inches thick, and 6 feet 3 inches high. How many cubic feet are its contents?

159. A room 42 feet long and 21 feet wide contains 7938 cubic feet of space. What is its height?

160. How many cubic feet in a cube of stone whose edges are each 5 feet 6 inches?

161. What is the value of a rectangular stone 8 feet long and 2 feet 6 inches square, at $1.25 a cubic foot?

162. A rectangular piece of timber whose contents are 103.53 cubic feet, is 24.65 feet long, and 2.80 feet wide. What is its depth?


4 feet.
8 feet.
4 feet.
1 CORD FOOT
4 FEET

160. A **Range of Wood** 8 feet long, 4 feet wide, and 4 feet high, is a *cord.*

A *cord foot* is 1 foot in length of this range.

A *perch* of masonry, or of building stone, is 24.75 cubic feet.

163. What is the value of a range of wood 20 feet long, 6 feet high, and 4 feet wide, at $4 a cord?

164. A range of wood is 4 feet wide and 4 feet high. How long must the range be to contain 2½ cords?

165. How high must a range of wood 20 feet long and 4 feet wide be piled to make 5 cords?

166. How many perch of stone in an underpinning wall that is 33 feet long, 1 foot 6 inches thick, and 2 feet high?

167. How many perch of stone masonry in a wall 66 feet long, 9 feet high, and 3 feet thick?

161. A **Board Foot,** which is 1 foot long, 1 foot wide, and 1 inch thick, is the unit for measuring *lumber,* or *sawed timber.* Hence,

To find the contents in board feet of a board, plank, joist, etc., *multiply the length and breadth, each in feet, by the thickness in inches.*

168. How many board feet in a board 18 feet long, 1 foot 6 inches wide, and 1 inch thick?

169. What are the contents in board feet of a plank 20 feet long, 2 feet wide, and 2½ inches thick?

170. What is the cost of a piece of sawed timber 16 feet long, 8 inches wide, and 6 inches thick, at 3 cents a board foot?

171. What cost 40 joists, each 16 ft. long, 4 in. wide, and 3 in. thick, at $20 per thousand?

172. What will $1\frac{1}{4}$ inch flooring cost at $25 per thousand, to floor a room 30 ft. by 28 ft. ?

173. How many cords of wood in 6 sticks of square timber, each 36 ft. long, and 16 in. wide?

174. What will a range of wood $\frac{1}{4}$ of a mile long, 4 ft. high, and 4 ft. wide, be worth at $ $7\frac{1}{2}$ per cord?

175. Find the cost of the following:

25 thousand shingles	@	$2.80
$16\frac{1}{2}$ thousand ft. spruce lumber	"	14.50
$8\frac{1}{2}$ thousand ft. hard pine	"	36.00

176. If $0.70 is paid for sawing a cord of wood; each stick into two pieces, how much should be paid for sawing a cord of wood, each stick into four pieces?

QUESTIONS.

134. What are linear measures? The table of linear measures?

136. What are square measures? The table of square measures?

138. What are cubic measures? The table of cubic measures?

139. What are liquid measures? The table of liquid measures?

140. What are dry measures? The table of dry measures?

141. What are avoirdupois weights? What is the table of avoirdupois weights?

142. What is time? What is the table of time?

146. What is reduction? 147. Reduction descending? 149. Reduction ascending?

153. What is a rectangle? 155. How is the area of a rectangle found?

158. What are the contents of a rectangular volume? 159. How are the contents of a rectangular volume found?

160. What range of wood is a cord? 161. What is a board foot?

REVIEW.

162. **1.** What part of a yard is 9 inches? 18 inches? 27 inches?

2. If 1 yard 3 quarters of cloth will make a vest, how many vests may be made from 4 yards 1 quarter?

3. Twelve of anything make a dozen. If you should sell 44 eggs at 15 cents a dozen, how much would they bring?

4. A path is 12 yards 2 feet long; what is its length in feet?

5. If you should buy a bushel of nuts for $ 2.50 and sell them at 10 cents a quart, how much would you gain?

6. A boy spent 16 days of his vacation by the seashore, the remainder, which was 12 days, at home. How many weeks was his vacation?

7. What is the value of 2 gallons 2 quarts of milk at 6 cents a quart?

8. Twenty of anything make a score. A man is 3 score 5 years old; how many years old is he?

9. John has sold 64 gills of nuts; how many pecks has he sold?

10. Susan bought at one time 8 ounces of candy, at another 1 pound 2 ounces, and at another 6 ounces. How much did she buy in all?

11. From a 5-gallon keg of molasses there have been drawn out 2 gallons 3 quarts. How much remains in the keg?

12. James has 3 lines, each 2 yards 2 feet in length. What is the entire length of the 3?

13. A farmer divided 5 pounds 4 ounces of maple sugar equally among 4 boys. How much did each boy receive?

14. How many days from April 28 to May 17?

15. If June 25 is on Monday, on what day will July 4 be?

16. Twelve dozen of anything make a gross. What will 48 dozen of buttons cost at \$ $1\frac{1}{4}$ a gross?

17. A range of wood is 16 feet long, 4 feet wide, and 4 feet high. What is it worth at \$ $5\frac{1}{2}$ a cord?

18. A rectangular board 20 feet long contains 50 board feet. What is its width?

WRITTEN EXERCISES.

19. How many pounds in 7 T. 18 cwt.?

20. How many tons in 15800 pounds?

21. How many grains in 2 lb. 11 oz. 20 gr.?

22. How many pounds in 16820 grains?

23. How many quarts in 32 hogsheads?

24. How many hogsheads in 8064 quarts?

25. How many pints in 27 hhds. 18 gal. 2 qts.?

26. In 13756 pints how many hogsheads?

27. How many quarts in 17 bu. 1 pk. 4 qt.?

28. How many bushels in 556 quarts?

29. In 12 rd. 5 yd. 2 ft. how many feet?

30. In 215 feet how many rods?

31. Reduce 3 A. 140 sq. rds. to square yards.

32. Reduce 18755 square yards to acres.

33. Reduce 3 cu. yd. 9 cu. ft. 1480 cu. in. to cu. in.

34. Reduce 157000 cubic inches to cubic yards.

35. Reduce 506 d. 22 h. 40 min. to minutes.

36. Reduce 730000 minutes to days.

37. How many minutes in 1 common year?

38. How many years in 525600 minutes?

39. In a barrel of flour there are 196 pounds. What cost $1\frac{1}{2}$ barrel of flour at 6 cents a pound?

40. In a barrel of pork there are 200 pounds. How much can be made in retailing, at 13 cents a pound, a barrel of pork, which cost $ 19.50?

41. How many hogsheads of molasses, at 12 cents a quart, can be bought for $ 75.12?

42. If the heart beats 70 times a minute, how many days will be required for it to beat 705600 times?

43. If a ship can sail at the rate of 8 miles an hour, how many weeks will be required for it to sail 1448 miles?

44. If 9 candles can be made from 1 pound of tallow, how many dozen can be made from 60 pounds?

45. How many minutes in an average year of 365 days 6 hours?

46. How many hours in a common year from January 12 to April 24?

47. If a man travels one day 30 m. 103 rd. and another day 27 m. 137 rd., how far does he travel in all?

48. If a merchant sells 11 yards 3 quarters from a piece of print containing 30 yards $1\frac{1}{2}$ quarters, how much will remain?

49. If, from a cask containing 64 gal. 3 qt. 1 pt. 2 gi. of molasses, there should leak 11 gal. 3 qt. 1 pt. 1 gi., how much would remain in the cask?

50. How many days from March 4, 1879, to April 1, 1881?

51. Multiply 24 d. 8 h. 42 min. 40 sec. by 15, using its factors 3 and 5?

52. Allowing 60 pounds to make a bushel of potatoes, how many pounds of potatoes, at 20 cents a peck, can be bought for $8.60?

53. How many times will a wheel 18 ft. 4 in. around, turn in going 100 miles?

54. How many loaves of bread can be made from 200 pounds of flour, allowing 10 ounces to a loaf?

55. How many shingles will be required to cover the side of a roof 30 feet by 20 feet, allowing 1 thousand to cover just 100 square feet?

56. How high must a range of wood, 40 feet long and 4 feet wide, be piled to make 10 cords?

57. How many days, of 10 hours each, may be saved for study in 10 years, of 365 days each, by rising 30 minutes earlier each day?

58. Allowing in a mow 500 cubic feet of space to a ton of hay, how many tons of hay will there be in a mow 20 feet long, 12 feet wide, and 15 feet high?

59. If a bushel of corn will occupy 8 tenths of a cubic foot of space, how many bushels will a bin contain whose inside measure is 6 ft. long, 4 ft. 6 in. wide, and 3 ft. deep?

60. If a man can lift 201 lb. 12 oz. and that is 8 times what a boy can lift, how much can a boy lift?

61. How many square yards of carpeting will it take for a room 26 ft. by 32 ft.

62. A milkman left 2 gal. 3 qt. 1 pt. of milk at a boarding-house every morning for 6 days. What quantity did he leave in that time?

63. How much must be paid for a range of wood 100 feet long, 4 feet wide, and 4 feet high, at $ 7.50 a cord?

64. Allowing 40 cubic feet for the storage of a ton of anthracite coal, how long must a bin be that is 4½ feet wide and 4 feet deep, to hold 2¼ tons?

65. Into how many lots, of 76 square rods each, can a field of 29 acres 40 square rods be divided, and what will remain?

66. If, when laid, 20 bricks are required to a cubic foot of brick-work, how many bricks will be required to build a wall 64 feet long, 10 feet high, and 16 inches thick?

67. What will $16\frac{2}{3}$ pounds cost if one pound costs $33\frac{1}{3}$ cents ?

68. Paid $ 100 for $14\frac{1}{6}$ yards of silk ; what was that per yard ?

69. Bought $3\frac{1}{4}$ pounds of steak at 28 cents a pound, and a dozen and a half of eggs at 30 cents a dozen. How much change should I receive from a $ 2 bill ?

70. Add $0.5\frac{1}{4}$, $0.18\frac{3}{4}$, and $0.56\frac{4}{5}$.

71. What will 3 T. 90 lb. 4 oz. sugar cost at a cent an ounce ?

72. What will it cost to pave a rectangular yard 100 ft. long by 60 ft. wide at 75 cents per square yard ?

73. If a cubic foot of granite weighs 166 pounds, what will a cube 5 feet long weigh ?

REVIEW QUESTIONS.

2. What is a number? 143. What is a denomination? 144. What is a denominate number? 145. What is a compound denominate number?

78. What is United States Money? 79. What are coins? 39. Why may cents be written as hundredths, and mills as thousandths, of a dollar?

152. What is a surface? 153. What is a rectangle? 135. What is a square?

156. What is a volume? 157. What is a rectangular volume? 137. What is a cube?

160. What is a cord of wood? What is a perch of masonry? 161. What is a board foot?

161. How do you find the contents in board feet of a board, plank, joist, etc.?

PERCENTAGE.

163. 1. What is $\frac{1}{100}$ of 100? Of 200? Of 300? Of 400?

2. What is $\frac{2}{100}$ of 100? $\frac{3}{100}$ of 100? $\frac{5}{100}$ of 100?

3. How many hundredths of $1 are 3 cents? 6 cents? 13 cents?

4. How many hundredths of 1 is expressed by .03? By .05? By .13? By .31?

5. How many hundredths of a dollar is $\frac{1}{4}$ of a dollar? $\frac{1}{2}$ of a dollar?

6. How many hundredths of anything is $\frac{1}{4}$ of it? $\frac{1}{2}$ of it? $\frac{1}{5}$ of it? $\frac{2}{5}$ of it?

7. In a school of 50 pupils, 3 were tardy? How many was that per hundred?

164. **Per Cent** means by the hundred. Thus, 3 per cent means 3 of every 100, or 3 hundredths.

165. The **Sign** % is used for the words *per cent.*

Thus, 3 % means 3 per cent, and $4\frac{1}{2}$ % means $4\frac{1}{2}$ per cent.

166. The **Rate** is the number of hundredths.

It may be expressed in the form of a common fraction, or of a decimal. Thus,

1 per cent, or 1 %, may be written $\frac{1}{100}$, or .01.
3 per cent, or 3 %, may be written $\frac{3}{100}$, or .03.
6 per cent, or 6 %, may be written $\frac{6}{100}$, or .06.
$12\frac{1}{2}$ per cent, or $12\frac{1}{2}$ % may be written $\frac{12\frac{1}{2}}{100}$, or $.12\frac{1}{2}$.
100 per cent, or 100 %, may be written $\frac{100}{100}$, or 1.00.
103 per cent, or 103 %, may be written $\frac{103}{100}$, or 1.03.

167. Any **Per Cent** of a number is such a part of the number as is denoted by the rate.

168. **Percentage** is the process of computing in hundredths.

EXERCISES.

Express decimally —

8. 4 %.
9. 7 %.
10. 9 %.
11. 10 %.
12. $10\frac{1}{2}$ %.
13. $7\frac{1}{2}$ %.
14. $\frac{1}{3}$ %.
15. 16 %.
16. 100 %.
17. 101 %.
18. 107 %.
19. $31\frac{1}{2}$ %.

Write in the form of a common fraction in its smallest terms —

20. 8 %.
21. 20 %.
22. 50 %.
23. $17\frac{1}{2}$ %.
24. 75 %.
25. 90 %.
26. 110 %.
27. 150 %.
28. 125 %.

29. What per cent of a number is $\frac{1}{4}$ of it?

Solution. — As any number is 100 per cent of itself, $\frac{1}{4}$ of the number must be $\frac{1}{4}$ of 100 per cent, or 25 per cent.

30. What per cent of a number is $\frac{1}{2}$ of it? $\frac{1}{5}$ of it?
31. What per cent of a number is $\frac{2}{5}$ of it? $\frac{3}{4}$ of it?
32. What per cent of a number is $\frac{3}{8}$ of it? $\frac{7}{10}$ of it?

To find any Per Cent of a Number.

33. What is 5 % of 600?

Solution. — As 5 % is $\frac{5}{100}$, 5 % of 600 is $\frac{5}{100}$ of 600. $\frac{1}{100}$ of 600 is 6, and $\frac{5}{100}$ is 5 times 6, which is 30.

34. What is 9 % of 100 yards? Of 300 yards?

35. What is 6 % of $12? 4 per cent of $25?

36. In a school of 300 pupils 25 % are in the first class. How many are in the first class?

WRITTEN EXERCISES.

37. What is 3 % of $225?

$$\begin{array}{r} \$225 \\ .03 \\ \hline \$6.75 \end{array}$$

Solution. — As 3 % is $\frac{3}{100} = .03$, 3 % of $225 is .03 of $225, which is $6.75.

38. What is 15 % of 184 tons?

39. What is 35 % of 172 gallons.

40. What is 5½% of 656 feet?

169. To find any per cent of a number:

Rule.

Multiply the given number by the rate per cent expressed decimally.

What is —

41. 5 % of $72?

42. 8 % of $90?

43. 6 % of $112?

44. 9 % of $38?

45. 3½ % of 164 yards?

46. 5¼ % of 8.75 bushels?

47. 7 % of 330 lb.

48. 20 % of 416 cubic feet.

49. A farmer raised 160 bushels of apples, and sold 80 % of the whole. How many did he sell?

50. Of $500, I have invested in the United States bonds 40 %, and the remainder in land. How much have I invested in stock, and how much in land?

51. From a deposit of $1800 in a bank, I have taken out 62 %; how much remains?

52. My carriage cost $250, and my horse 60 % as much. How much did my horse cost?

To find the Per Cent one Number is of another.

53. What per cent of 12 is 3?

Solution. — As 1 is $\frac{1}{12}$ of 12, 3 is $\frac{3}{12}$ of 12; and $\frac{3}{12} = \frac{1}{4} = .25$, or 25 %.

54. What per cent of 35 yards are 7 yards?

55. If the cost of raising corn is 9 bushels for every 27 grown, what is the cost per cent?

56. What per cent of $28 is $7?

57. If you should spend $6 for every $50 earned, what per cent of your earnings would you spend?

WRITTEN EXERCISES.

58. What per cent of 128 yards are 16 yards?

$$\begin{array}{r l} & .12\tfrac{1}{2}\text{, or } 12\tfrac{1}{2}\ \% \\ 128) & 16.00 \\ & \underline{12\,8} \\ & 3\,20 \\ & \underline{2\,56} \\ & \frac{64}{128} = \frac{1}{2} \end{array}$$

Solution. — 16 yards are $\frac{16}{128}$ of 128 yards. As $\frac{16}{128} = .12\frac{1}{2}$, 16 yards $= .12\frac{1}{2}$, or $12\frac{1}{2}$ % of 128 yards.

59. What per cent of $124 is $18.60?

60. What per cent of 160 miles are 8 miles?

170. To find the per cent one number is of another:

Rule.

Divide the number denoting the per cent by that on which the per cent is reckoned, and extend the division to hundredths.

What per cent is —

61. 6 of 50.

62. 7 of 140.

63. 9 of 150.

64. 15 of 90.

65. 7 lb. of 56 lb.

66. $12 of $60.

67. 25 men of 225.

68. 21 miles of 280 miles.

69. If American gold coin is 9 parts pure gold to 1 part alloy, what per cent is alloy?

70. If a miller takes 4 quarts toll for grinding 32 quarts, what per cent is the toll?

71. A farmer's flock of 120 sheep was increased to 156. What was the increase per cent.

72. I have a farm of 160 acres; 120 acres of it is in pasture. What per cent of the whole is that?

73. John is 12 years old, and his father 54 years. What per cent is John's age of that of his father?

To Find a Number when a Per Cent of it is given.

74. 25 is 5 % of what number?

Solution. — As 5 % of the number is 25, 1 % of it is $\frac{1}{5}$ of 25, or 5; 100 %, or the number, must be 100 times 5, or 500.

75. Of the money I had given me, I have left 3 %, which is $ 30. What sum had been given me?

76. A farmer sold 32 bushels of wheat, which was 8 % of his crop. How much was his crop?

77. Mary has 24 peaches, which is 12 % of the number which grew upon a tree. What number grew upon the tree?

78. A boy sold some oranges for 15 % more than cost, and made 45 cents. What was the cost of the oranges?

WRITTEN EXERCISES.

79. $ 35.50 is 25 % of what number?

$ 35.50 ÷ .25 = $ 142.

Or,

```
       $142
  25) $3550
       25
       ---
       105
       100
       ---
         50
         50
```

Solution.—As $ 35.50 is 25 % of the number, 1 % of it is $\frac{1}{25}$ of $ 35.50, which is $ 1.42, and 100 %, or the number, is 100 times $ 1.42, or $ 142.

Or as $ 35.50 is 25 %, or .25, of the number, that number must be $\frac{1}{.25}$ of $ 35.50, or, making divisor and dividend integers (Art. 130), $\frac{1}{25}$ of $ 3550, which is $ 142.

80. 120 is 75 % of what number?

81. $ 7.20 is 8 % of what number?

171. To find a number when a per cent of it is given:

Rule.

Divide the per cent by the given rate.

Of what is —

82. 132, 6 % ?

83. 384, 12 % ?

84. 96, 15 % ?

85. $175, 8 % ?

86. 28.15 yd. 5 % ?

87. 12.11 feet, 35 % ?

88. Jane gave for a bonnet $ 15, which was 60 % of the cost of her cloak. What was the cost of her cloak ?

89. Paid $ 90 for a horse, which was 75 % of the cost of a carriage. What was the cost of the carriage ?

APPLICATIONS OF PERCENTAGE.

90. Sold a watch which cost me $ 40, at a profit of 20 %. How much did I gain ?

Solution. — As 20 % is .20, or $\frac{1}{5}$, the gain was $\frac{1}{5}$ of $ 40, which is $ 8.

91. Bought flour at $ 6 a barrel, and sold it at a profit of 25 %. How much was gained on a barrel ?

92. Sold tea which cost 50 cents a pound at a loss of 10 %. What was the loss a pound ?

93. Sold a horse, which cost $ 120, at a profit of 10 %. For how much was he sold ?

Solution. — As the horse was sold at a profit of 10 %, he was sold at 10 % above $ 120. 10 % of $ 120 is $ 12, and 10 % above $ 120 is $ 120 + $ 12, which is $ 132.

94. A merchant wishes to get a profit of 25 % on coffee which cost him 32 cents a pound. What must be his selling price a pound ?

95. If you have a book which cost you $ 4.40, at what price must you sell it to gain $12\frac{1}{2}$ % ?

96. Sold a watch which cost me $40 at a profit of $8. What rate was the profit?

Solution. — As $8 is $\frac{8}{40}$ of $40, and $\frac{8}{40} = \frac{1}{5}$ or .20, the rate was 20 %.

97. Sold flour which cost $8 a barrel at a profit of $2 a barrel. What rate was the profit?

98. Sold tea which cost 50 cents at a loss of 5 cents. What rate was the loss?

99. Sold a horse which cost $120 for $132. What rate was the gain?

100. By selling a barrel of flour at 25 % above cost I gained $2. What was the cost?

Solution. — As 25 % is .25 or $\frac{1}{4}$, the gain was $\frac{1}{4}$ of the cost, and if $2 is $\frac{1}{4}$ of the cost, 4 times $\frac{1}{4}$, or the cost, must be 4 times $2, which is $8.

101. By selling butter at 10 % below cost, I lost 4 cents a pound. What was the cost?

102. If by selling a horse at 25 % above cost, I gained $30, what was the cost?

103. If I lose 20 % by selling cloth at $1 a yard below cost, what was the cost a yard?

WRITTEN EXERCISES.

104. A house cost me $3500. What can I gain by selling it at a profit of $4\frac{1}{2}$ %?

105. A merchant sells goods, which cost him $1648 at a profit of 5 %. How much did he gain?

106. If stocks bought for $300 should advance $9\frac{1}{2}$ %, what would be the gain?

107. If a cask of molasses which contained 120 gallons should lose 5 % of its contents by leakage, how much would be the loss?

108. Bought a horse for $ 216, and sold him at a loss of 10 %. How much was the loss?

109. The profits of a business of $ 12650 are $8\frac{1}{2}$ %. How much are the profits?

110. Bought wood for $ 6, and gained in selling it $ 0.72. What was the rate of gain?

111. When molasses is bought for 56 cents a gallon and is sold for 63 cents, what is the rate of gain?

112. Bought a horse for $ 125 and sold it for $ 12.60 below cost. What was the rate of loss?

113. If I sell a farm that cost me $ 5000 for $ 4700, what is the rate of loss?

114. By selling a house at 6 % below cost, I lost $ 300. What was the cost?

115. A merchant by selling goods at 10 % above cost made $ 500. What was their cost?

116. Sold a horse 20 % above cost and made $ 51. What was the cost of the horse?

QUESTIONS.

164. What does per cent mean? 166. What is the rate? 167. What is any per cent of a number? 168. What is percentage?

169. How do you find any per cent of a number? 170. How do you find the per cent one number is of another? 171. How do you find a number when a per cent of it is given?

INTEREST.

172. 1. When 6 % is required for the use of money, how many cents must be paid for the use of $1 for a year? How many dollars for the use of $100 for a year?

2. At 5 % a year, what should be paid for the use of $1 for 2 years? For 3 years?

3. At 7 % a year, what should be paid for the use of $100 for 1 year? For 6 months, or $\frac{1}{2}$ of a year? For 2 years?

4. When money is loaned at 6 % a year, how many hundredths of the money is the rate?

5. When 6 % is paid for the use of $100 for a year, what is that for two months, or $\frac{1}{6}$ of a year? For 1 month, or $\frac{1}{12}$ of a year?

6. If $200 is borrowed for 2 years at 7 % a year, how much is due the lender at the end of the time?

7. At 4 % what decimal part of the money borrowed is taken for its use?

173. **Interest** is the money paid for the use of money.

174. The **Principal** is the sum for whose use interest is paid.

175. The **Rate** of interest is the number of hundredths of the principal taken for its use one year.

176. The **Amount** is the sum of the principal and the interest.

177. The **Interest** of any sum, at 6 per cent, —

For 12 months, or 1 year, is .06 of the principal
" 2 months, or $\frac{1}{6}$ year, is .01 "
" 1 month, or $\frac{1}{12}$ year, is .005 "
" 6 days, or $\frac{1}{5}$ month, is .001 "
" 1 day, or $\frac{1}{30}$ month, is $.000\frac{1}{6}$ "

WRITTEN EXERCISES.

8. What is the interest on $ 212 at 6 % for 2 years 7 months 15 days?

$ 212.
$.157\frac{1}{2}$
106
1484
1060
212
$ 33.390

Solution. — 2 years 7 months 15 days = 31.5 months.

As the interest at 6 % is half as many hundredths of the principal as there are months in the time, it must be one half of 31.5 hundredths of $ 212, or $.157\frac{1}{2}$ of $ 212, which is $ 33.39.

NOTE. — In the answer to questions in interest, it is considered to be sufficiently exact to reject mills in the result if less than 5, and if 5, or more than 5, to call them 1 cent. The result is then said to be to the nearest cent.

9. What is the amount of $ 654 at 6 % for 93 days?

$ 654
$.015\frac{1}{2}$
327
3270
654
$ 10.137
654.
$ 664.14

Solution. — As the interest at 6 % is one sixth as many thousandths of the principal as there are days in the time, it must be one sixth of .093, or $.015\frac{1}{2}$ of $ 654, which is $ 10.137, or, to the nearest cent, $ 10.14. Adding the principal, $ 654, to this interest we have as the amount $ 664.14.

10. Find the interest of $896 for 2 y. 7 mo. 23 d. at 6 %.

Solution.

2 y. 7 mo. 23 d. = 31 mo. 23 d.

$$\begin{array}{rl} \frac{1}{2} \text{ of } .31 & = .155 \\ \frac{1}{6} \text{ of } .023 & = \underline{.003\frac{5}{6}} \\ & .158\frac{5}{6} \end{array}$$

$$\begin{array}{r} \$\,896 \\ \underline{.158\frac{5}{6}} \\ 6)\,\underline{4480} \\ 746\frac{2}{3} \\ 7168 \\ 4480 \\ \underline{896} \\ \$\,142.314\frac{2}{3},\text{ Ans.} \end{array}$$

11. What is the amount of $664, at 6 % interest for 2 years 6 months.

178. To compute 6 % interest:

Rule.

Multiply the principal by one half as many hundredths as there are months, or by one sixth as many thousandths as there are days, in the time.

To find the amount add the principal and the interest.

What is the interest at 6 %, of

12. $187.25 for 1 y. 4 mo.

13. $98.20 for 1 y. 28 d.

14. $126.46 for 9 mo.

15. $318 for 10 mo. 16 d.

16. $426.50 for 3 y. 7 m. 18 d.

17. $500 for 93 days.

18. $620 for 2 y. 3 mo. 3 d.

19. $715 for 1 y. 5 mo. 10 d.

20. $812 for 2 y. 9 mo. 18 d.

21. $750 for 3 y. 2 mo. 21 d.

22. What is the amount of $263.48 for 2 mo. 21 d. at 6 % ?

179. When the rate is other than 6 % : *Find the interest at 6 per cent, and increase or diminish this interest by such a part of itself as will give the interest at the required rate.* Thus,

To find 4 % interest subtract ⅓ of 6 % interest from itself.
" 4½ " subtract ¼ " " from itself.
" 5 " subtract ⅙ " " from itself.
" 7 " add ⅙ " " to itself.
" 7½ " add ¼ " " to itself.
" 8 " add ⅓ " " to itself.

23. What is the interest of $640 for 3 years 7 months, at 7 % ?

$$\begin{array}{r} \$\,640 \\ .21\tfrac{1}{2} \\ \hline 320 \\ 640 \\ 1280 \\ \hline 6)\ \$\,137.60 \\ 22.966+ \\ \hline \$\,160.566 \end{array}$$

Solution. — 3 years 7 months = 43 months.

The interest of $640 at 6 % is .21½ of $640, which is $137.60. This interest increased by one sixth of itself, or $137.60 + $22.966 +, is, to the nearest cent, $160.57, which is the interest at 7 %.

24. What is the interest of $780 for 5 years, at 7 % ?

25. What is the interest of $30.50 for 3 years, at 5 % ?

26. What is the interest of $130.60 for 6 months at 7 % ?

27. What is the interest of $130.60 for 8 months, at 7 % ?

28. What is the interest of $516 for 123 days at 7 % ?

29. What is the interest of $234.92 for 33 days, at 7 % ?

30. What is the interest of $420 for 63 days, at 9 % ?

31. What is the interest of $226 for 2 years 3 months at 5 % ?

32. What is the interest of $210 for 4 years 5 months at 7 % ?

33. What is the interest of $67.42 for 1 year 7 months 15 days, at 6 % ?

34. What is the interest of $333 for 5 years 2 months 5 days, at 8 % ?

35. What is the amount of $54.98 for 6 months 20 days, at 7 % ?

36. What is the amount of $45.15 for 1 year 4 months 9 days, at 5 % ?

37. What is the amount of $100 from May 16, 1879, to August 31, 1881, at 10 % ?

QUESTIONS.

173. What is interest? 174. What is the principal? 175. What is the rate? 176. What is the amount?

177. What part of the principal is the interest at 6 per cent for 1 year? For 2 months? For 6 days?

178. How do you compute interest at 6 per cent? How is amount found? 179. How is the interest found when the rate is other than 6 per cent?

GENERAL REVIEW.

180. 1. If 5 oranges cost 10 cents, how much will 12 oranges cost?

2. If 7 yards of cloth cost $ 28, how much will 9 yards cost?

3. If 8 pounds of sugar cost 72 cents, what will 6 pounds cost?

4. If 5 quarts of milk cost 40 cents, what will 7½ quarts cost?

5. When $ 6 will buy 24 yards of cloth, how many yards will $ 8 buy?

6. If 9 pounds of veal cost 54 cents, how many pounds can be bought for 42 cents?

7. If 7 plows can be bought for $ 77, how many can be bought for $ 44?

8. When $ 8 will buy 40 rakes, how many can be bought for $ 11?

9. If 12 men can do a piece of work in 6 days, how many days will it take 9 men to do the same?

Solution.—If 12 men can do a piece of work in 6 days, it will take 1 man 12 times 6 days, or 72 days, to do it, and 9 men ⅑ of 72 days, or 8 days.

10. If 7 men can dig a ditch in 8 days, how many days will be required for 14 men to dig it?

11. If 6 horses eat a bag of oats in 10 days, how many horses will eat the same quantity in 15 days?

12. How long will it take 5 men to reap a field, if it takes 15 men 3 days to reap it?

13. How long will it take 9 men to build a wall, if it takes 6 men 6 days to build it?

14. If 7 books cost $21, what will 11 books cost?

15. When 8 barrels of flour will pay for 12 yards of broadcloth, at $6 a yard, how much is the flour per barrel?

16. If 5 men can hoe a field of corn in 9 days, in what time can 15 men hoe it?

17. When 10 oranges will pay for 20 apples, at 2 cents each, what is the price of each orange?

18. When 10 bushels of wheat will pay for 5 cords of wood, at $4 per cord, how much is the wheat per bushel?

19. $\frac{1}{2}$ of a pole is above ground, and 3 feet is $\frac{1}{4}$ of the part above ground. What is the length of the pole?

20. If a ship can sail at the rate of 10 miles per hour, how far will it sail in $\frac{3}{8}$ of a day?

21. At 50 cents a foot, what will a rod of iron fence cost?

22. From a pile of wood containing 6 cords of wood, there were sold 2 loads each 3 feet high, 4 feet wide, and 4 feet long. How much remained?

23. A piece of land is 40 rods square. How many more square rods does it contain than two pieces each 20 rods square?

24. A farmer sells his corn at 48 cents a bushel, which is 80% of what he sold for last year. What did he sell for last year?

25. What per cent does a grocer make by selling his goods at $\frac{4}{5}$ of their cost?

26. My price for a horse, which is held at 25% above cost, is $100. If I should sell him at 25% off my price, how much should I lose?

27. What is the interest on $120, for 2 years 3 months at 6%?

28. If you should lend $6 at 7%, how much will be due you at the end of 2 years 6 months?

WRITTEN EXERCISES.

29. A merchant receives from the sale of coffee $519.60, of tea $310.20, and of sugar $640.45. How much does he receive from the whole?

30. If you should buy a farm for $5600, and sell a part of it for $3780, and the remainder for $2340, how much would you make by the operation?

31. The difference of two numbers is 397, and the smaller number is 532; what is the larger number?

32. A and B together had $655, but A having lost $65, they then had equal sums; how much had B?

33. A merchant bought 55 bales of cloth, each bale containing 36 pieces, and each piece 30 yards; how many yards did he buy in all?

34. Two travelers set out from towns 363 miles distant and travel towards each other till they meet, when it appears one has traveled 93 miles farther than the other; how far had each traveled?

35. If the product of two numbers is 1960, and one of the numbers is 35, what is the other number?

36. If I have $ 3642, and purchase 404 barrels of flour at $ 9 a barrel, how much shall I have left after paying for the flour?

37. How many bushels of wheat, at $ 1.25 a bushel, will pay for 10 yards of broadcloth, at $ 4.50 per yard?

38. Divide the product of 35 × 20 by 640 − 515.

39. If 5 tons of hay cost $ 106.25, what will 13 tons cost?

40. In what time will 48 men do a piece of work that 12 men can do in 24 days?

41. When 15 pounds of sugar can be bought for $ 1.80, what will 18 pounds cost?

42. How far can a man travel in 18 days, at the rate of 378 miles in 6 days?

43. If a man travels 1134 miles in 18 days, how many miles will he travel in 6 days?

44. How many pounds of coffee can be bought for $ 21, if 30 pounds cost $ 6?

45. When 18 pounds of sugar can be bought for $ 2.16, what will 15 pounds cost?

46. If 9 acres of grass will pasture 15 cows, how many cows will 6 acres pasture?

47. When 141 tons of coal can be bought for $ 775.50, how much is the cost of 100 tons?

48. Required the prime factors of 1365.

49. Reduce $312\frac{4}{5}$ to an equivalent improper fraction.

50. Reduce 1564 to an equivalent fractional form.

51. Change $\frac{3}{5}$ and $\frac{5}{7}$ to equivalent fractions having a common denominator.

52. A merchant sold at one time $\frac{2}{5}$ of a ship, at another time $\frac{1}{2}$ of the remainder; what part of the whole ship did he sell at both times?

53. $\frac{2}{5} + \frac{3}{10}$ subtracted from 1 leaves how many?

54. What is the cost of 6 acres and 80 rods of land, at $ 12.50 an acre?

55. What is the difference of time between Jan. 30, 1865, and May 16 of the same year?

56. How many $1\frac{1}{2}$ pint bottles can be filled from a cask of wine containing 54 gal. 1 qt. 1 pt.?

57. Gave $\$\,4\frac{5}{8}$ for a vest, $\$\,10\frac{3}{4}$ for a coat, and $\$\,5\frac{3}{16}$ for boots; how much did the whole cost?

58. At $\$\,\frac{3}{8}$ a yard, how many yards of cloth can be had for $ 12?

59. What decimal is equivalent to $\frac{15}{16}$?

60. What is the difference between thirty thousandths and thirty thousand?

61. Purchased two loads of hay, the first weighing 1689 lb., and the second 1761 lb.; what was the weight of the two loads in tons?

62. If 105 is $\frac{3}{7}$ of some number, what is that number?

63. When $5500 is the value of $\frac{1}{2}$ of $\frac{5}{8}$ of a cotton factory, what is the whole of it worth?

64. How long will 2295 pounds of beef last a garrison of 45 men, if each man should get 1 pound 3 times a week?

65. A farmer, being asked how many sheep he had, answered, that he had them in 4 fields; in the first he had $\frac{1}{4}$ of his flock, in the second $\frac{1}{5}$, in the third $\frac{1}{6}$, and in the fourth 46; how many had he in all?

66. What common fraction is equivalent to .9375?

67. What per cent of 60 is 15?

68. What is the quotient, when 500 is the dividend and .05 the divisor?

69. Required the quotient of .05 divided by 500.

70. What is the product of .0001 multiplied by 500?

71. At $8 a thousand, what cost 19340 bricks?

72. What cost 42555 feet of boards at $40 a thousand?

73. The sum of two numbers is 93, and one of them is $11\frac{1}{5}$; what is the other?

74. If I buy corn for 60 cents a bushel, and sell it at a loss of 20 %, what do I get a bushel for it?

75. Of a certain pole, 5 feet stands in earth, 10 feet in water, and $\frac{2}{5}$ above water; what is the length of the pole?

76. What fraction is that to which, if you add $\frac{5}{7}$, the sum will be $\frac{27}{28}$?

77. How many rectangular blocks, $1\frac{1}{2}$ feet wide and 2 feet long, will pave a cellar floor 15 feet wide and 16 feet long?

78. When the dividend is 17.28, and the quotient 14.4, what is the divisor?

79. When the dividend is .001, and the quotient .000001, what is the divisor?

80. Bought a horse, and paid $72 down, which was $\frac{6}{11}$ of the price; what was the price?

81. What is the interest on $600 for 6 months, at $7\frac{3}{10}$ per cent?

82. Bought a cask of molasses, containing 65 gallons, for $26; for how much per gallon must I sell it to gain $12\frac{1}{2}$ per cent?

83. What is the value of a pile of wood 36 feet long, 4 feet wide, and 6 feet high, at $6 per cord?

84. How many steps of 2 feet 6 inches each, must a man take, in walking a mile?

85. If the cost of $\frac{7}{8}$ of a farm is $3500, what is $\frac{3}{4}$ of it worth?

86. Find the amount of the following bill:

BURLINGTON, Dec. 17, 1881.

James Bruce

Bought of Amos J. Smith & Co.

16 bbl. New Process Flour	@	$6.50
50 bu. Winter Wheat	"	1.05
75 bu. No. 1 White Oats	"	.42
		$

87. If $\frac{3}{8}$ of a cord of wood cost $2.85, what will $\frac{3}{4}$ of a cord cost?

88. If $4\frac{1}{2}$ pounds of butter are worth $1.44, what are $11\frac{7}{16}$ pounds worth?

89. A man sold some property that cost him $1350 at a profit of 14 %. How much did he get for it?

90. Bought a horse for $112 and sold him for $129.92. What was the rate per cent of gain?

91. What is the interest of $750 for 1 y. 6 mo. 15 d. at 7 %?

92. How much wood is contained in a load 8 feet long, $3\frac{1}{2}$ feet wide, and $5\frac{1}{2}$ feet high?

93. If $\frac{7}{8}$ of a ship cost $35000, what is $\frac{1}{2}$ of it worth?

94. How many yards of cloth $1\frac{1}{4}$ yards wide will line 16 yards $\frac{5}{8}$ of a yard wide?

95. Bought a horse for $125, but not proving as good as expected, I sold him for $110; what was the loss per cent?

96. Bought a house for $3600, and sold it at a loss of 7 %. How much did I obtain for it?

97. What is the value of a small rectangular farm, 72 rods long and 40 rods wide, at $50 per acre?

98. What is the interest on a note of $500 from Jan. 28 to May 21, 1881, at 7 %?

99. What is the interest of $1560 from June 1, 1880, to Aug. 15, 1883, at 5 %.

100. What is the amount of $250, at 6 % interest, from May 16, 1880, to Jan. 1, 1882?

101. What is the amount of $1300, at 8 % interest, from April 9, 1879, to July 1, 1881?

HSMT

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